

# Osnove matematične analize

## Vaje, 12. teden

1. \* Poišči minimum in maksimum funkcije  $f(x, y) = xy$  na krogu

$$x^2 + y^2 \leq 1.$$

Rešitev: V notranjosti kroga ni ekstremov (točka  $(0, 0)$  je sedlo). Na krožnici sta minimuma v  $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$  in  $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ , maksimuma sta v  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$  in  $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ .

2. Izračunaj naslednje nedoločene integrale:

(a) \*  $\int 3x^2 - 5x + 1 \, dx$ ,

(b) \*  $\int \frac{2+x}{1-x} \, dx$ ,

(c)  $\int (1 + \frac{1}{x^2}) \sqrt{x}\sqrt{x} \, dx$ ,

(d)  $\int \frac{3}{x-8} \, dx$ ,

(e) \*  $\int \frac{x}{(x-1)(x-8)} \, dx$ ,

(f)  $\int \frac{1}{x^2-2x+2} \, dx$ ,

(g) \*  $\int \frac{1}{x(\log x)^2} \, dx$ ,

(h)  $\int \tan x \, dx$ ,

(i)  $\int \frac{e^x}{e^x-1} \, dx$ ,

(j)  $\int x e^{-(x^2+1)} \, dx$ ,

(k)  $\int \sin(3x) \, dx$ ,

(l)  $\int \frac{\arcsin x}{\sqrt{1-x^2}} \, dx$ ,

(m) \*  $\int x^3 \log x \, dx$ .

(n)  $\int \frac{\log x}{x^2} \, dx$ .

Rešitve: (a)  $x^3 - \frac{5}{2}x^2 + x + c$ , (b)  $-x - 3 \log |1-x| + c$ , (c)  $\frac{4}{7}x^{\frac{7}{4}} - 4x^{-\frac{1}{4}} + c$ ,  
(d)  $3 \log |x-8| + c$ , (e)  $-\frac{1}{7} \log |x-1| + \frac{8}{7} \log |x-8| + c$ , (f)  $\arctan(x-1) + c$ ,  
(g)  $-\frac{1}{\log x} + c$ , (h)  $-\log |\cos(x)| + c$ , (i)  $\log |e^x - 1| + c$ , (j)  $-\frac{1}{2} e^{-(x^2+1)} + c$ ,  
(k)  $-\frac{1}{3} \cos(3x) + c$ , (l)  $\frac{(\arcsin x)^2}{2} + c$ , (m)  $\frac{1}{4} x^4 \log x - \frac{x^4}{16} + c$ , (n)  $-\frac{\log x}{x} - \frac{1}{x} + c$ .

3. Izračunaj določene integrale

(a) \*  $\int_0^\pi x \sin(3x) \, dx$

(b)  $\int_{-\pi}^\pi \cos(x) \sin^2(x) \, dx$

(c) \*  $\int_e^{e^2} \frac{(\log x)^2 - 2 \log x}{x} \, dx$

(d) \*  $\int_0^2 \frac{e^x}{e^{2x}+1} \, dx$

(e)  $\int_0^{\sqrt{\log 2}} x e^{-x^2} \, dx$

(f)  $\int_0^2 x e^{-x} \, dx$

(g)  $\int_{-1}^2 \frac{x}{x^2-x-6} \, dx$

Rešitve: (a)  $\frac{\pi}{3}$ , (b) 0, (c)  $-\frac{2}{3}$ , (d)  $\arctan(e^2) - \frac{\pi}{4}$ , (e)  $\frac{1}{4}$ , (f)  $1 - \frac{3}{e^2}$ , (g)  $-\frac{2 \log 2}{5}$ .

4. Izračunaj ploščine likov, ki jih omejujejo dane krivulje

(a) \*  $y = x^2 + 2x$  in  $y = x + 2$ .

(b)  $y = x^3 - x^2 + x$  in  $y = 3x$ .

(c) \*  $y^2 = 2x + 1$  in  $y = x - 1$ .

(d)  $y = \sin(x)$  in  $y = \cos(2x)$  na intervalu  $[\frac{\pi}{6}, \frac{5\pi}{6}]$ .

Rešitve: (a)  $\frac{9}{2}$  (b)  $\frac{37}{12}$  (c)  $\frac{16}{3}$  (d)  $\frac{3\sqrt{3}}{2}$