Topological Data Analysis
Lab work, 1st week

1. Define $d\colon \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ as

$$d((x_1,y_1),(x_2,y_2)) = \begin{cases} \|(x_1,y_1)-(x_2,y_2)\|_2, & (0,0),(x_1,y_1),(x_2,y_2) \text{ are collinear,} \\ \|(x_1,y_1)\|_2 + \|(x_2,y_2)\|_2, & \text{otherwise.} \end{cases}$$

   Draw the open balls

   (a) $B((0,0),1)$,

   (b) $B((3,0),4)$ and

   (c) $B((1,1),\sqrt{2})$.

2. Given the points $A(3,-4)$ and $B(4,3)$ in $\mathbb{R}^2$ find the parametrization for least three different paths

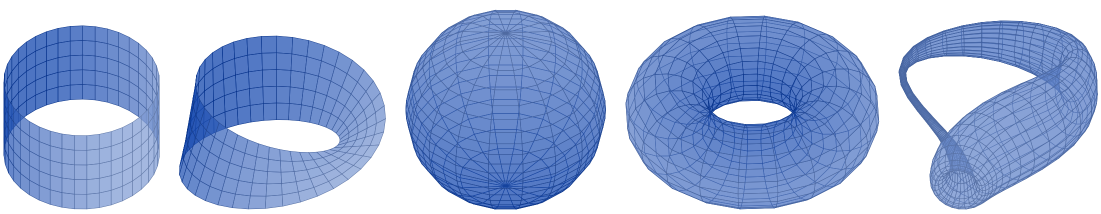
$$\alpha,\beta,\gamma\colon [0,1] \to \mathbb{R}^2$$

   from A to B.

3. Let $X = \{(x,y,z) \in \mathbb{R}^3\,;\, z^2 = x^2+y^2,\ 0<z<1\}$ and $Y = S^1 \times (0,1)$. Show that $X \cong Y$.

4. Let $X_n = S^n \setminus \{(0,\dots,0,1),(0,\dots,0,-1)\} \subset \mathbb{R}^{n+1}$ and $Y_n = S^{n-1} \times (-1,1) \subset \mathbb{R}^{n+1}$. Draw X_n and Y_n for $n = 0,1,2$. Prove that X_2 and Y_2 are homeomorphic.

5. Which of the following surfaces (cylinder, Moebius strip, sphere, torus, Klein bottle) are homeomorphic? Are any of them homotopy equivalent? If so, which? If not, why not?



6. Draw $X_n = S^{n-1}$ and $Y_n = \mathbb{R}^n \setminus \{0\}$ for $n = 1,2$. Show that X_2 and Y_2 are homotopy equivalent.

7. Show that the Moebius band $M = [-1,1]\times[-1,1]/_\sim$, where $(-1,y) \sim (1,-y)$ for all $y \in [-1,1]$, is homotopy equivalent to the circle $S^1 = [-1,1]/_\sim$, where $-1 \sim 1$.