We’ll finish the last week’s exercise and then solve the exercise below.

1. Let

\[ A = \begin{bmatrix}
-1 & 1 & 2 \\
1 & -1 & -2 \\
1 & -1 & 2 \\
-1 & 1 & -2
\end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix}
3 \\
-3 \\
2 \\
0
\end{bmatrix}. \]

(a) Find at least one generalized inverse of the matrix \( A \), i.e. a matrix \( G \) such that \( AGA = A \).

(b) Is the system \( Ax = b \) solvable? Find the orthogonal projection \( b' \) of the vector \( b \) onto \( C(A) \) and find all the solutions of the system \( Ax = b' \).

(c) Find the singular value decomposition of \( A \); \( A = USV^T \). This can be obtained using the eigenvalue decomposition of \( A^T A \).

(d) Find the Moore–Penrose pseudoinverse of \( A \) and compute \( A^+b \). Explain the result.

(e) Solve the exercise in octave, using the commands \( \text{svd}(A) \) and \( \text{pinv}(A) \).