1. The curve $K$ is parametrized by $\mathbf{r}(t) = [x(t), y(t)]^T = [t^3 - 4t, t^2 - 4]^T$.

(a) Find the intersections of the curve with the coordinate axes $x$ and $y$.
(b) Write down the equation of the tangent to $K$ at $t = 1$.
(c) Find the points where the tangents are parallel to the coordinate axes.
(d) Is there a point of self-intersection on $K$?
(e) Sketch the curve $K$.
(f) Evaluate the area of the region bounded by the loop of the curve $K$.

2. The *lemniscate* is a curve given in polar coordinates by

$$ r(\phi) = a \sqrt{\cos 2\phi}. $$

Write the parametrisation for the lemniscate and evaluate the area of one of the regions enclosed by a loop.

3. Evaluate the length of the curve $K$ given by

$$ \mathbf{p}(t) = [t^2 \cos t, t^2 \sin t]^T, \ t \in [0, 2\pi]. $$

4. Evaluate the length of one of the arcs of the cycloid given by

$$ \mathbf{q}(t) = [t - \sin t, 1 - \cos t]^T, \ t \in [0, 2\pi]. $$

What is the area between the $x$-axis and one arc of the cycloid? (A *cycloid* is a curve traced by a point on the rim of a wheel rolling along the $x$-axis. The parametrisation given above is for a circle with radius $r = 1$.)

5. **The circumference and area of a polygon.** A polygon $P$ in $\mathbb{R}^2$ is determined by a sequence of points $A_1, A_2, \ldots, A_k$. Write the functions $l = \text{obseg}(A)$ and $p1 = \text{ploscina}(A)$ that return the circumference and area of the polygon $P$. The polygon is given by the matrix

$$ A = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \end{bmatrix}. $$

Additional task: Both functions should verify that the points $A_1, A_2, \ldots, A_k$ do indeed represent a polygon.

6. A surface in $\mathbb{R}^3$ is given by the implicit equation

$$ \left( R - \sqrt{x^2 + y^2} \right)^2 + z^2 = r^2, $$

where $R > r$ are two positive numbers.
(a) Verify that
\begin{align*}
x(\phi, \theta) &= (R + r \cos \theta) \cos \phi \\
y(\phi, \theta) &= (R + r \cos \theta) \sin \phi \\
z(\phi, \theta) &= r \sin \theta
\end{align*}
is a parametrisation of this surface.

(b) For $R = 2$ in $r = 1$ find the equation of the tangent plane at the point $T(1, \sqrt{3}, 1)$ using two different approaches: Using the implicit equation and using the parametrisation.