1. The curve K is parametrized by $\mathbf{r}(t) = [x(t), y(t)]^\mathsf{T} = [t^3 - 4t, t^2 - 4]^\mathsf{T}$.
   (a) Find the intersections of the curve with the coordinate axes x and y.
   (b) Write down the equation of the tangent to K at $t = 1$.
   (c) Find the points where the tangents are parallel to the coordinate axes.
   (d) Is there a point of self-intersection on K?
   (e) Sketch the curve K.

2. Evaluate the length of the curve K given by

$$\mathbf{p}(t) = [t^2 \cos t, t^2 \sin t]^\mathsf{T},\ t \in [0, 2\pi].$$

3. Evaluate the length of one of the arcs of the cycloid given by

$$\mathbf{q}(t) = [t - \sin t, 1 - \cos t]^\mathsf{T},\ t \in [0, 2\pi].$$

   What is the area between the x-axis and one arc of the cycloid? (A *cycloid* is a curve traced by a point on the rim of a wheel rolling along the x-axis. The parametrisation given above is for a circle with radius $r = 1$.)

4. The *lemniscate* is a curve given in polar coordinates by

$$r(\phi) = a\sqrt{\cos 2\phi}.$$

   Find a parametrisation of the lemniscate and evaluate the area of one of the regions enclosed by a loop.

5. **The circumference and the area of a planar polygon.** A polygon P in $\mathbb{R}^2$ is determined by a sequence of points $A_1, A_2, \ldots, A_k$. Write Octave functions `l = circumference(A)` and `pl = area(A)` that return the circumference and the area of the polygon P. The polygon is given by a matrix

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \end{bmatrix}.$$

   Additional task: Both functions should verify that the points $A_1, A_2, \ldots, A_k$ do indeed represent a polygon.