University of Ljubljana, Faculty of Computer and Information Science

Swarm intelligence

Prof Dr Marko Robnik-Šikonja

Analysis of Algorithms and
Heuristic Problem Solving
Version 2025

Nature inspired methods

- Besides evolutionary computation, nature is an inspiration for many other computational algorithms.
- Swarm intelligence (SI) is the collective behavior of decentralized, self-organized systems, natural or artificial.
- A population of simple agents interacting locally with one another and with their environment.
- The agents follow very simple rules, and although there is no centralized control structure dictating how individual agents should behave, local, and to a certain degree random, interactions between such agents lead to the emergence of "intelligent" global behavior, unknown to the individual agents.
- Examples in natural systems of SI include ant colonies, bird flocking, animal herding, bacterial growth, fish schooling and microbial intelligence.

Computational SI

- Computational properties
  - Fixed population
  - Autonomous individual
  - Communication between agents
- We will cover
  - Particle swarm optimization
  - Ant colony optimization

Swarming – the definition

- Aggregation of similar animals, generally cruising in the same direction

- Termites swarm to build colonies
- Birds swarm to find food
- Bees swarm to reproduce

Swarming is powerful

- Swarms can achieve things that an individual cannot

Human swarms

Powerful … but simple

All evidence suggests:

- No central control
- Only simple rules for each individual
- Emergent phenomena
- Self-organization

Harness this power out of simplicity

- Technical systems are getting larger and more complex
  - Global control hard to define and program
  - Larger systems lead to more errors
- Swarm intelligence systems are:
  - Robust
  - Relatively simple (How to program a swarm?)

Swarming – example

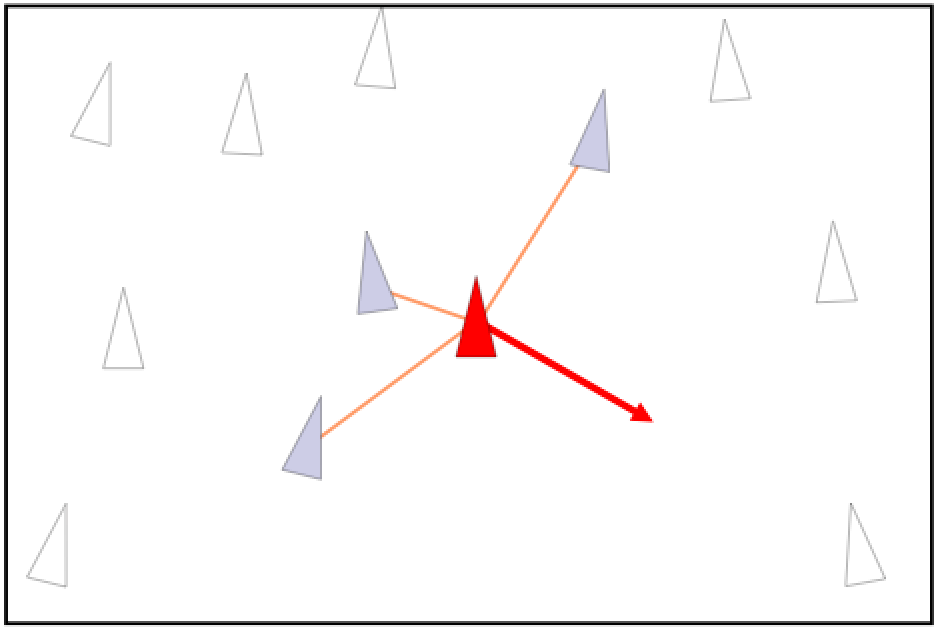
- Bird flocking

- "Boids" model was proposed by Reynolds (1985)
  - Boids = Bird-oids (bird like)

- Only three simple rules

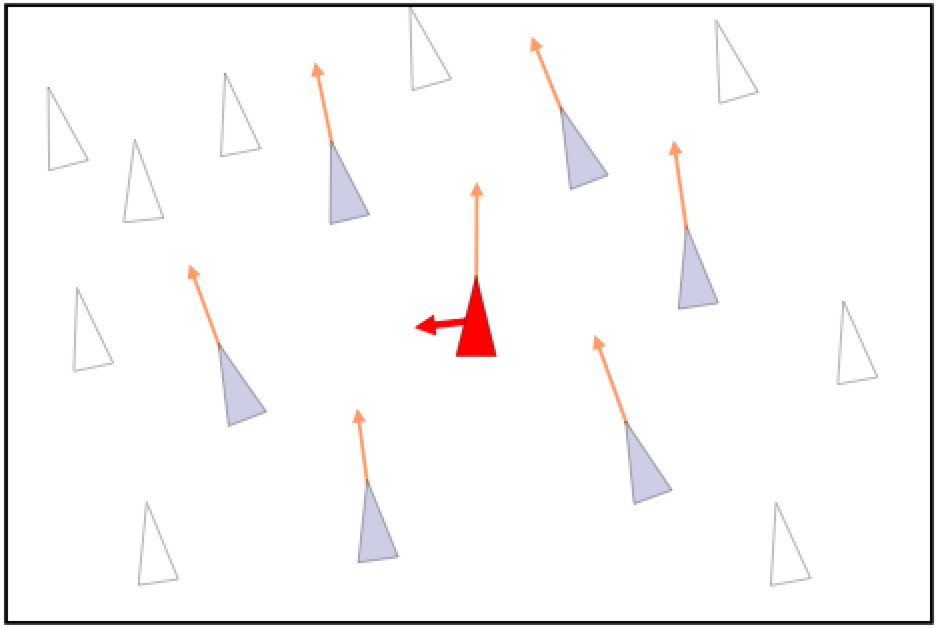
Collision Avoidance

- Rule 1: Avoid Collision with neighboring birds



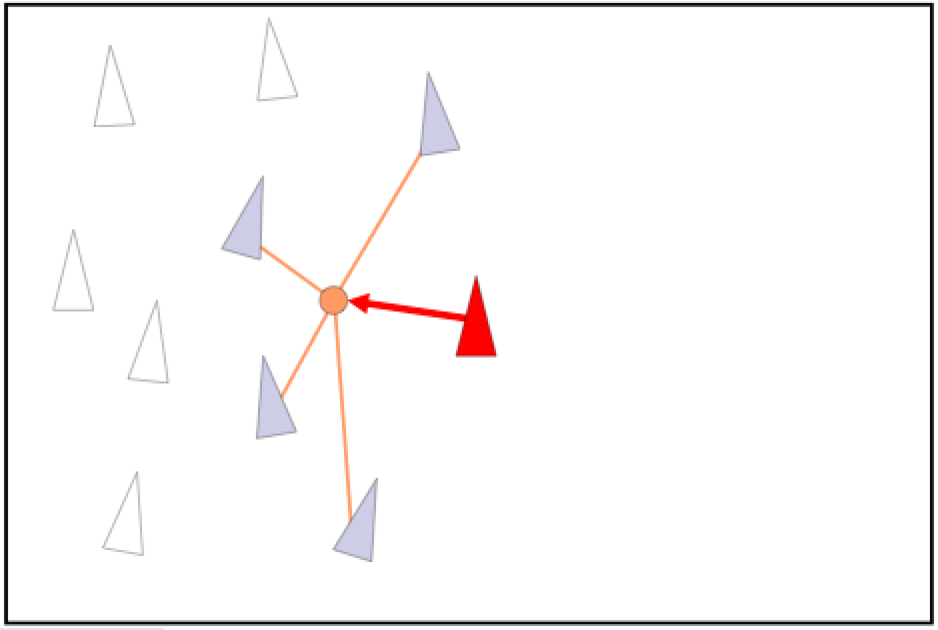
Velocity matching

- Rule 2: Match the velocity of neighboring birds



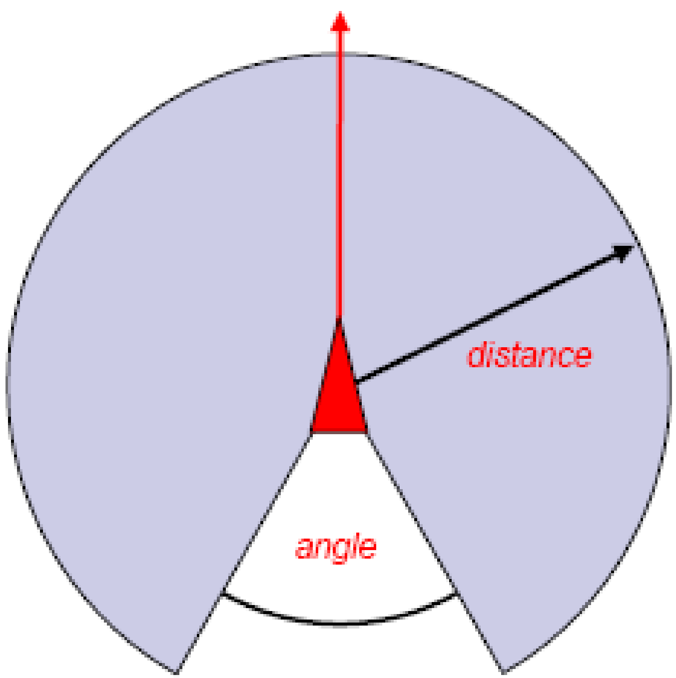
Flock centering

- Rule 3: Stay near neighboring birds



Define the neighborhood

- Model the view of a bird
- Only local knowledge, only local interaction
- Affects the swarm behavior (fish vs. birds)



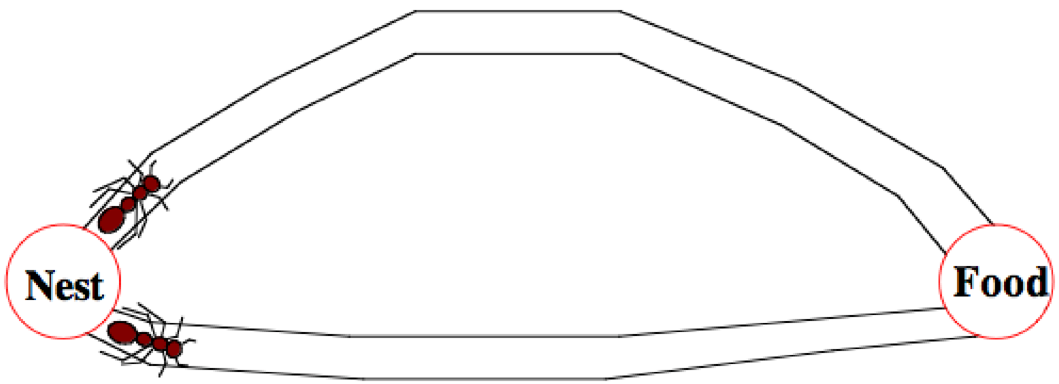
Swarming - characteristics

- Simple rules for each individual
- No central control
  - Decentralized and hence robust
- Emergent
  - Performs complex functions

Ant Colony Optimization - Biological Inspiration

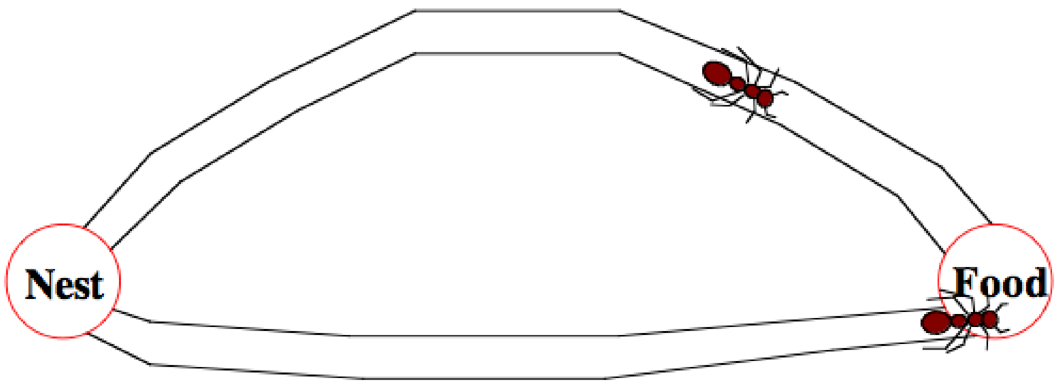
- Inspired by foraging behavior of ants.
- Ants find shortest path to food source from nest.
- Ants deposit pheromone along traveled path which is used by other ants to follow the trail.
- This kind of indirect communication via the local environment is called stigmergy.
- Has adaptability, robustness and redundancy.

Foraging behavior of Ants



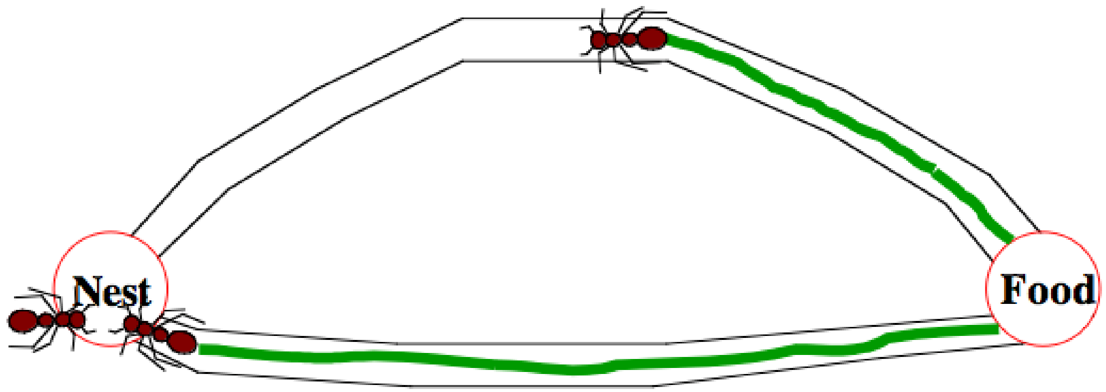
- 2 ants start with equal probability of going on either path.

Foraging behavior of Ants



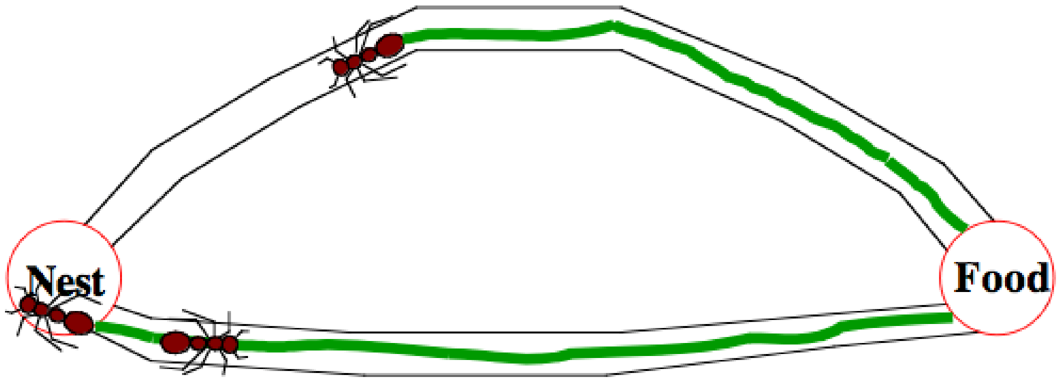
- The ant on shorter path has a shorter to-and-fro time from it's nest to the food.

Foraging behavior of Ants



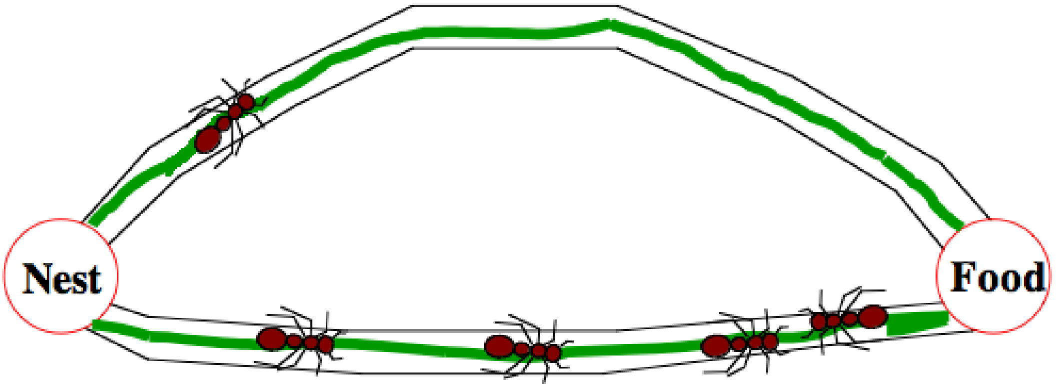
- The density of pheromone on the shorter path is higher because of 2 passes by the ant (as compared to 1 by the other).

Foraging behavior of Ants



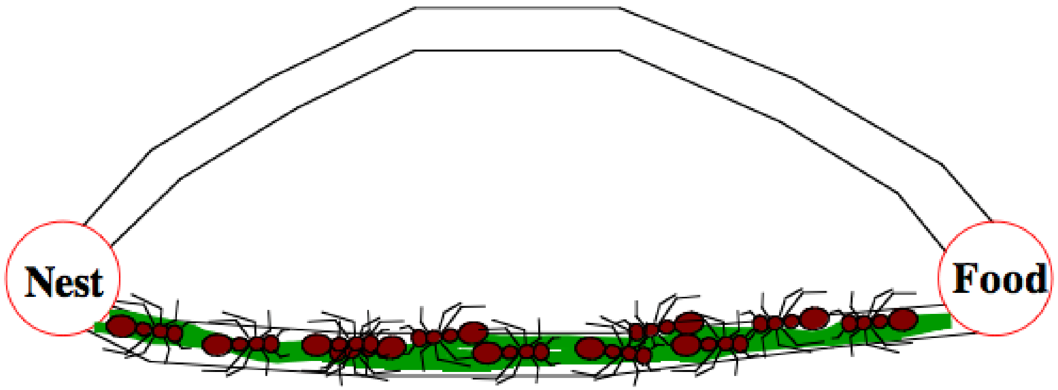
- The next ant takes the shorter route.

Foraging behavior of Ants



- Over many iterations, more ants begin using the path with higher pheromone, thereby further reinforcing it.

Foraging behavior of Ants



- After some time, the shorter path is almost exclusively used.

Ant colony

- Pheromones
- Ants lead their sisters to food source
- Evaporation
- Moving targets

Illustration of the dynamic adaptation

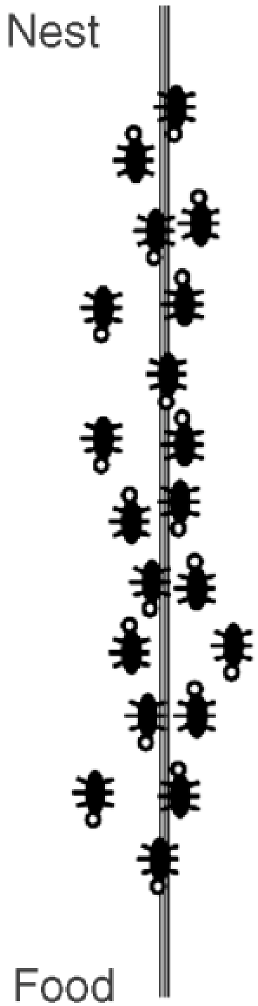


Illustration of the dynamic adaptation

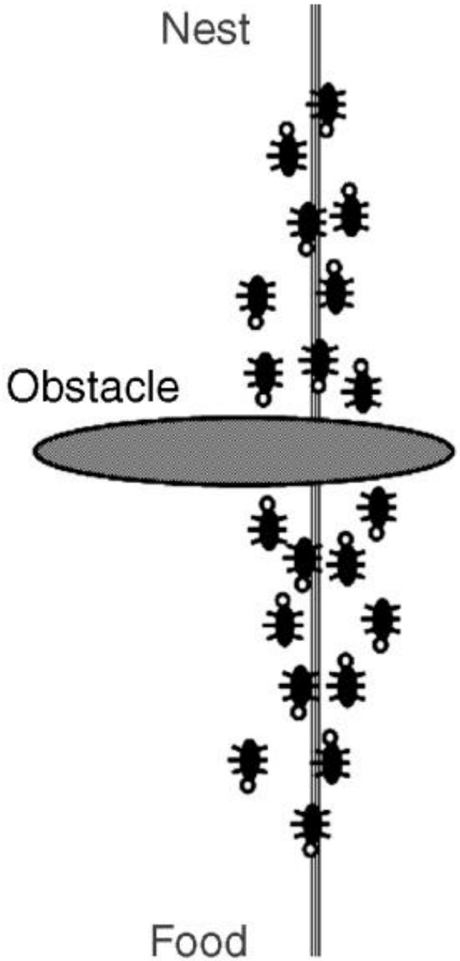


Illustration of the dynamic adaptation

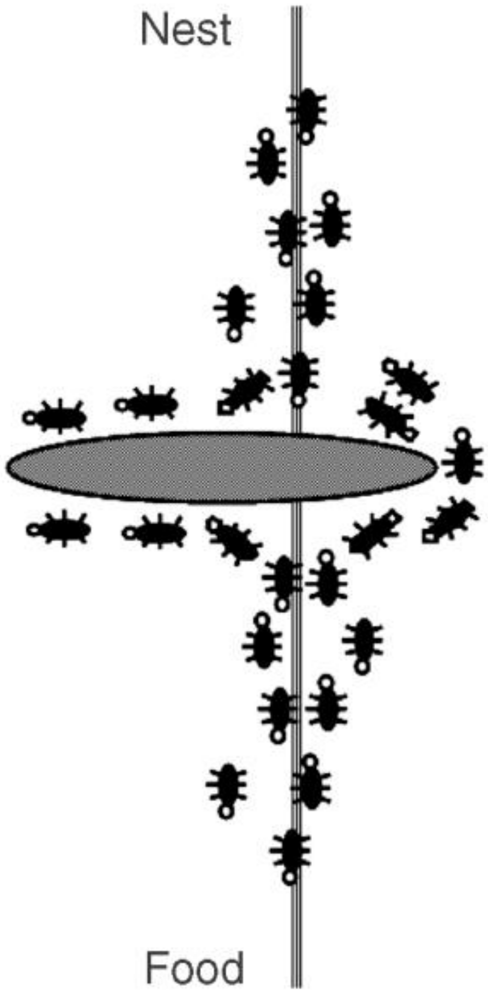


Illustration of the dynamic adaptation

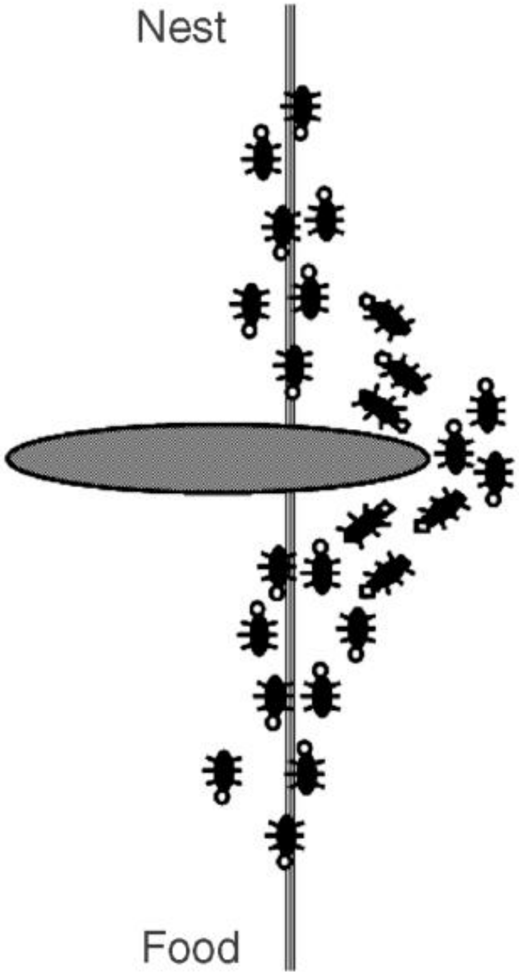
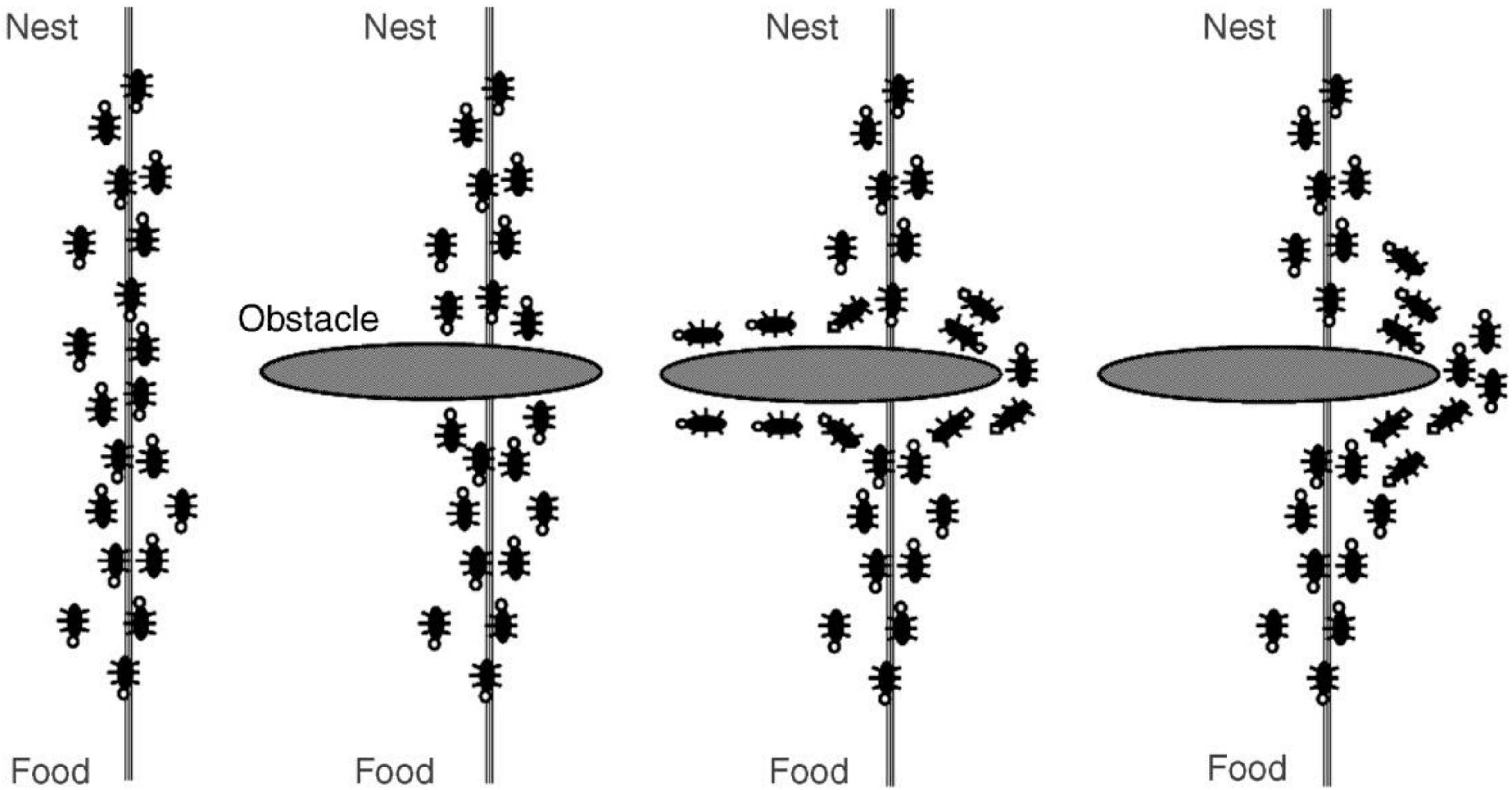


Illustration of the dynamic adaptation



Generic ACO

- Formalized into a metaheuristic.
- Artificial ants build solutions to an optimization problem and exchange info on their quality vis-à-vis real ants.
- A combinatorial optimization problem reduced to a construction graph.
- Ants build partial solutions in each iteration and deposit pheromone on each edge.

ACO pseudo code

```
Initialization of pheromones
do {
   for each  ant
      find solution: use pheromones and cost of path to select
    route
      apply local optimization (optional)
      update pheromones: enforcement, evaporation
} while (! satisfied)
return best overall solution
```

ACO details

- Pheromones updates
  - ρ speed of evaporation
- Trails updates
- Many variants

$$\tau_{i,j} = (1-\rho)\tau_{i,j} + \Delta\tau_{i,j}$$

$$\Delta\tau_{i,j} = \begin{Bmatrix} 1/C & \text{if ant takes the connection between i,j} \\ 0 & \text{otherwise} \end{Bmatrix},$$

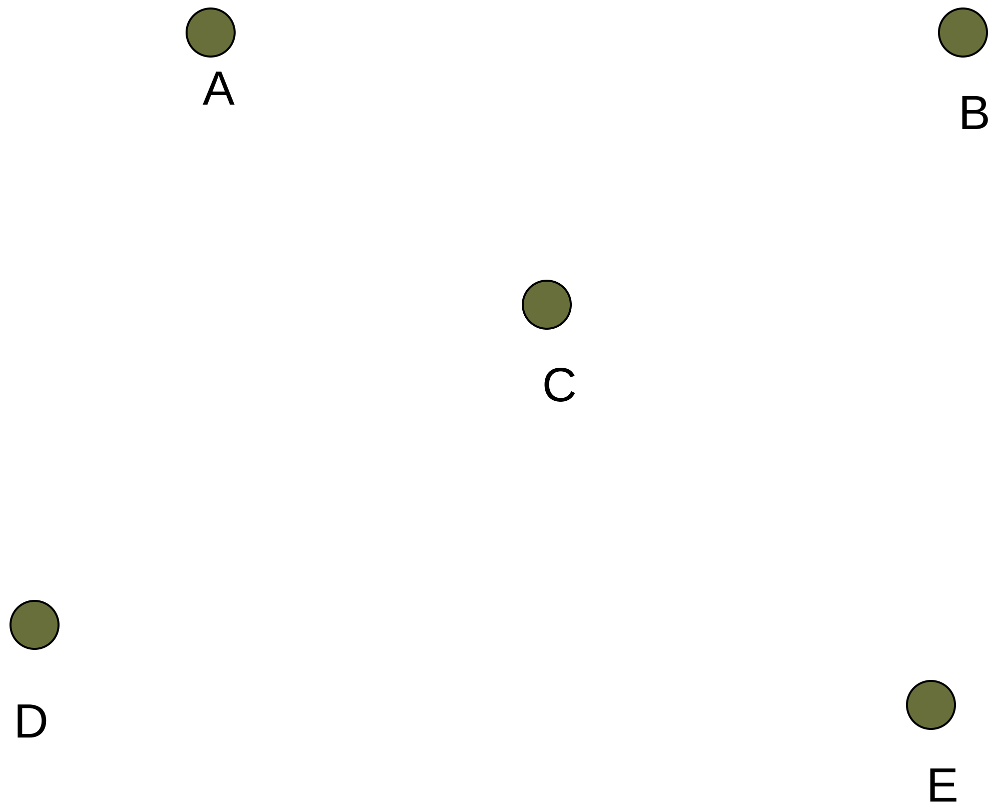
where C is a cost of edge i,j

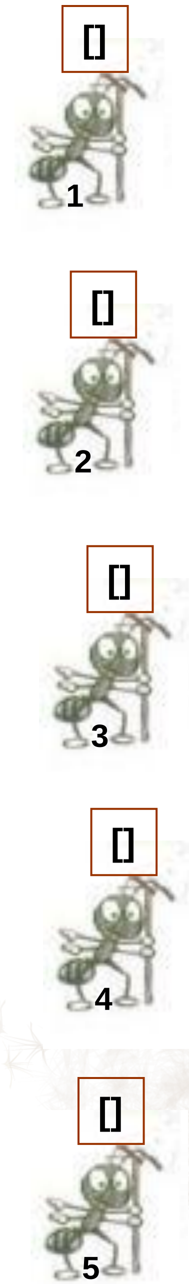
ACO for TSP

- Cities 1,2,…,n
- Cost $c_{i,j}$
- Construct the cheapest Hamiltonian tour through cities
- Attractiveness $\eta_{i,j} = 1/ c_{i,j}$
- Probability of ant's transition
  - α - impact of pheromones
  - β - impact of transition cost

$$p_{i,j} = \frac{\tau_{i,j}^{\alpha}\eta_{i,j}^{\beta}}{\sum \tau_{i,j}^{\alpha}\eta_{i,j}^{\beta}}$$

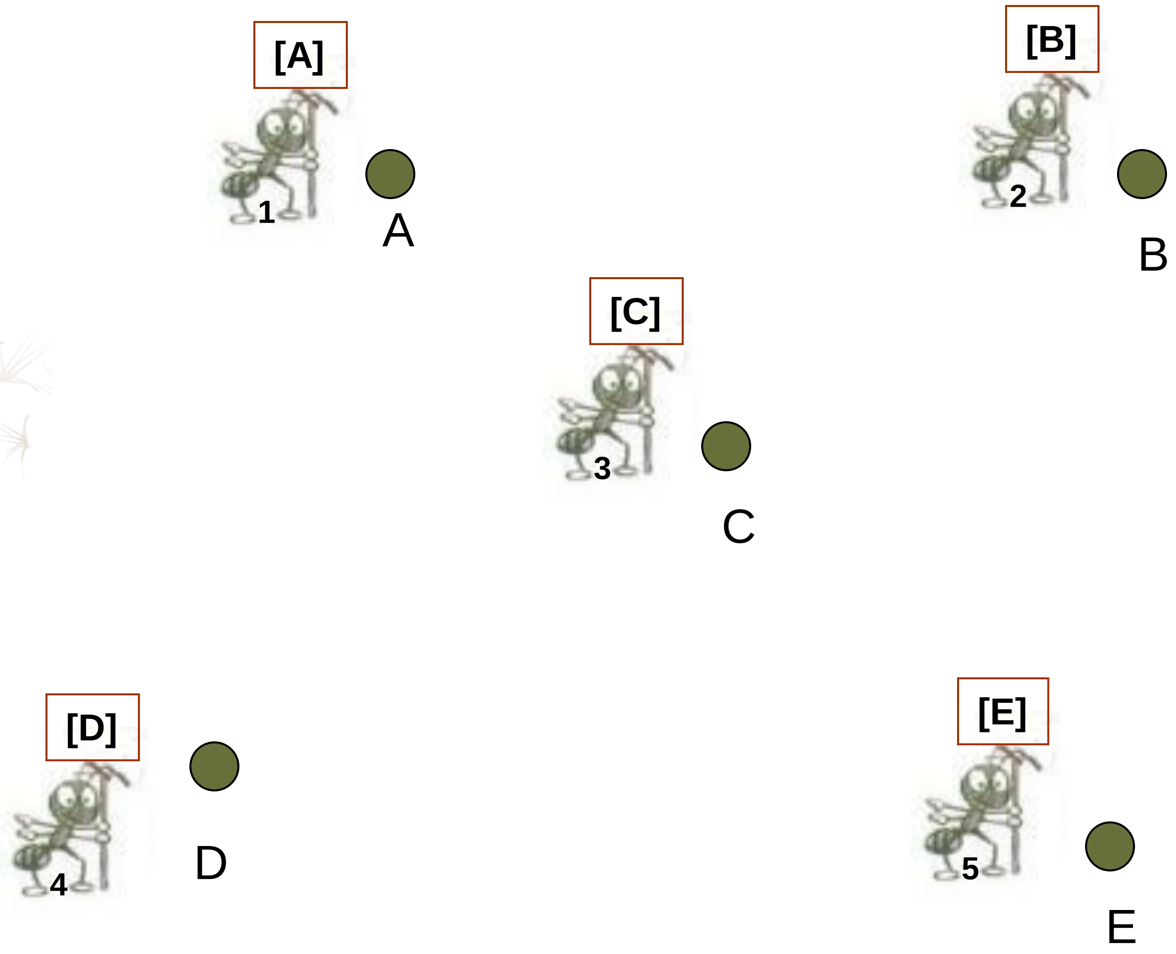
A simple TSP example



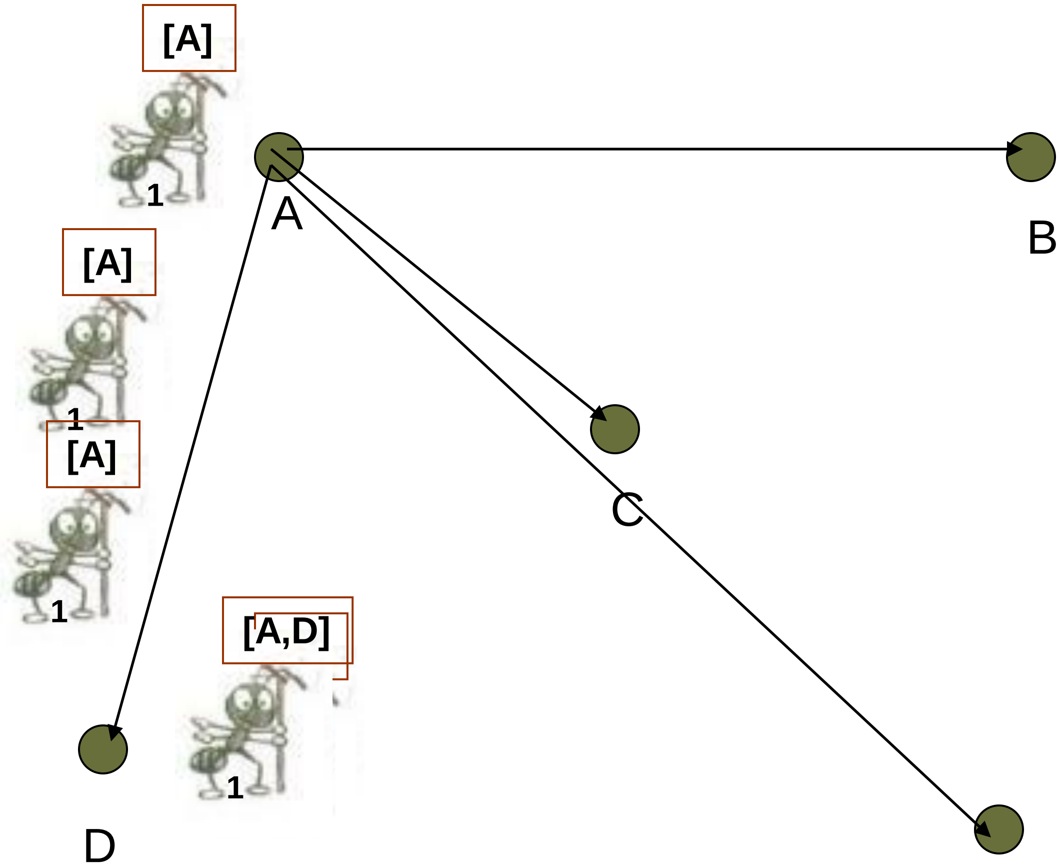


$d_{AB}=100; d_{BC}=60\ldots; d_{DE}=150$

Iteration 1

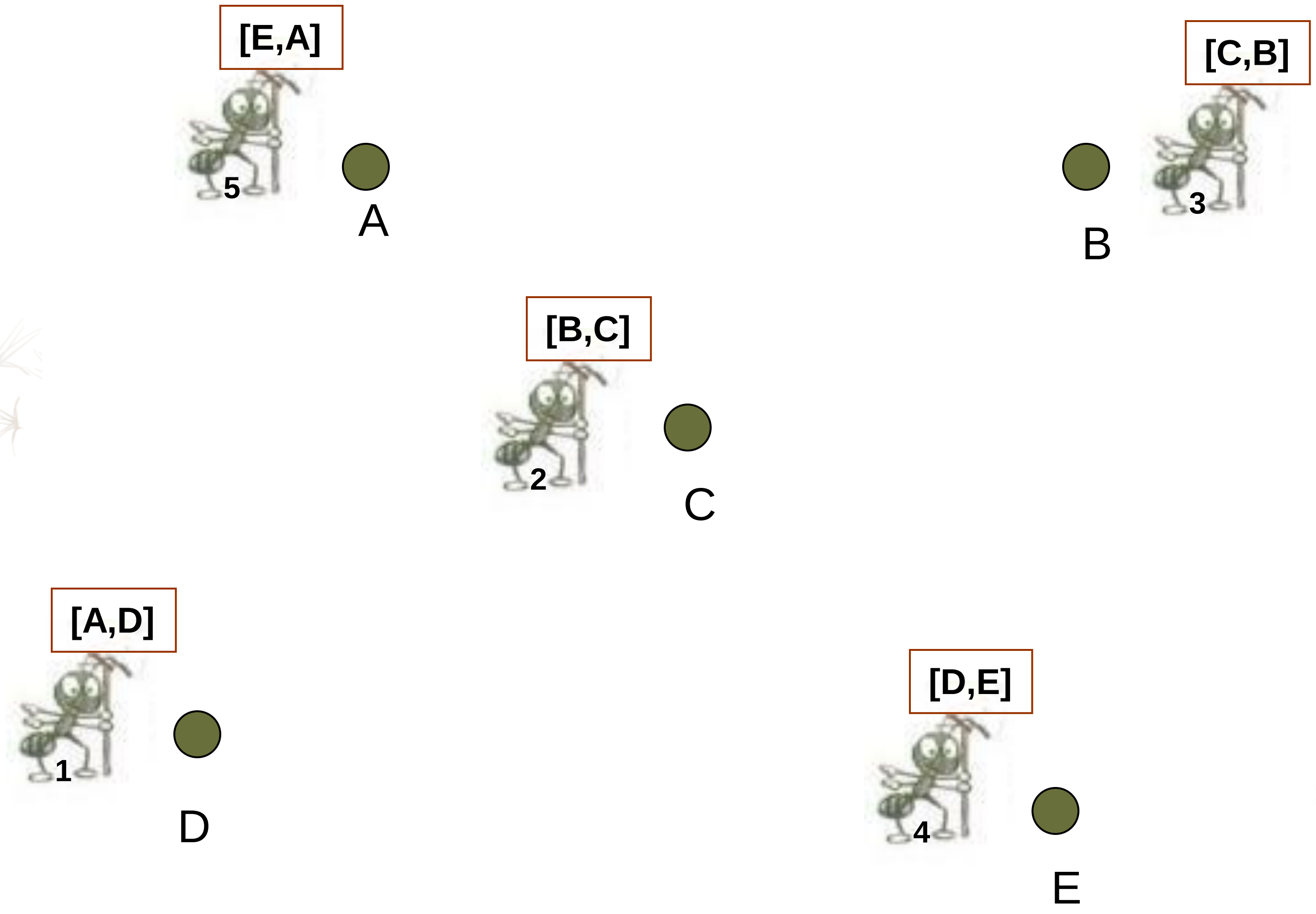


How to build next sub-solution?

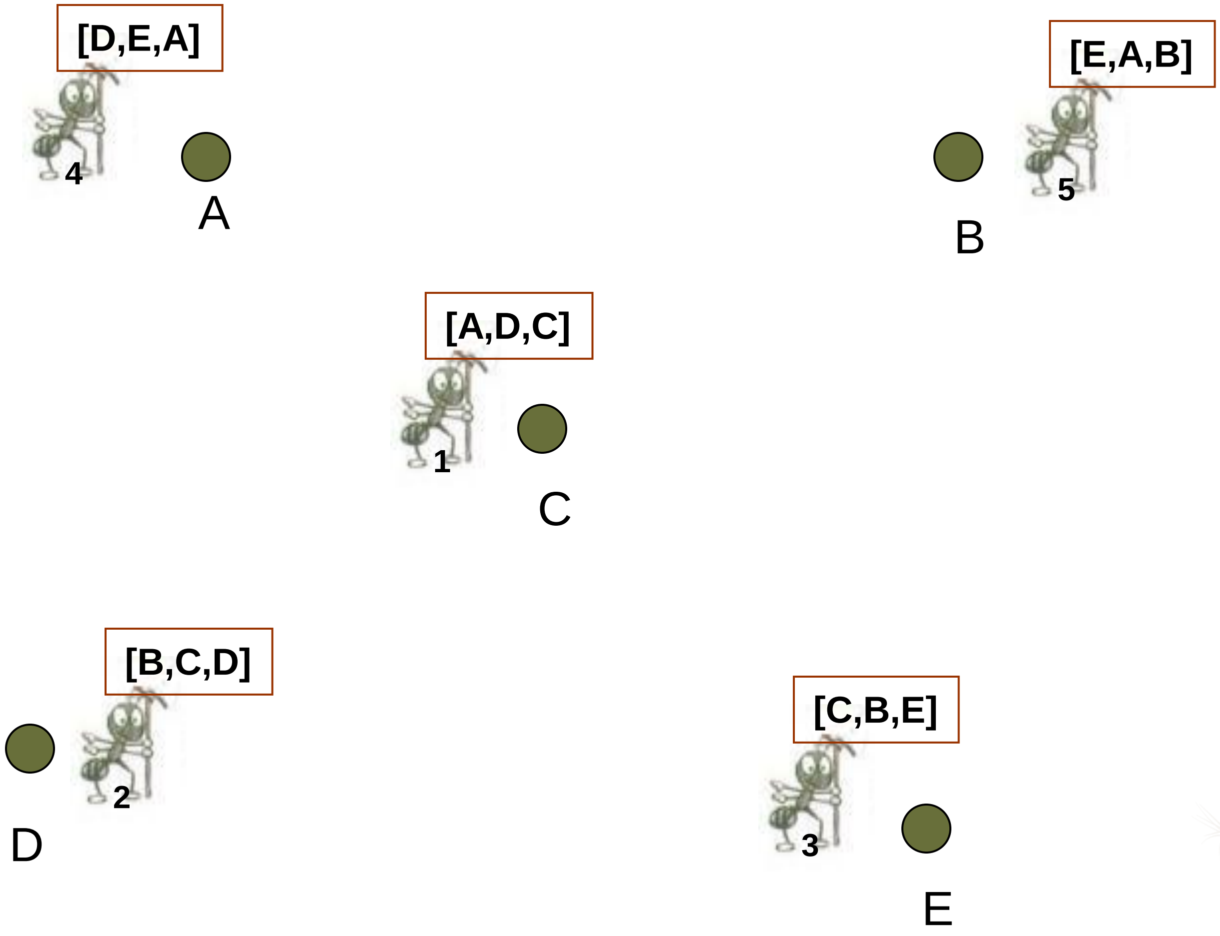


$$p_{ij}(t) = \begin{cases} \dfrac{\tau_{ij}(t)^{\alpha}\eta_{ij}{}^{\beta}}{\sum_{k\in allowed_k} \tau_{ik}(t)^{\alpha}\eta_{ik}{}^{\beta}} & \text{if j} \in \text{ allowed} \\ 0 & \text{otherwise} \end{cases}$$

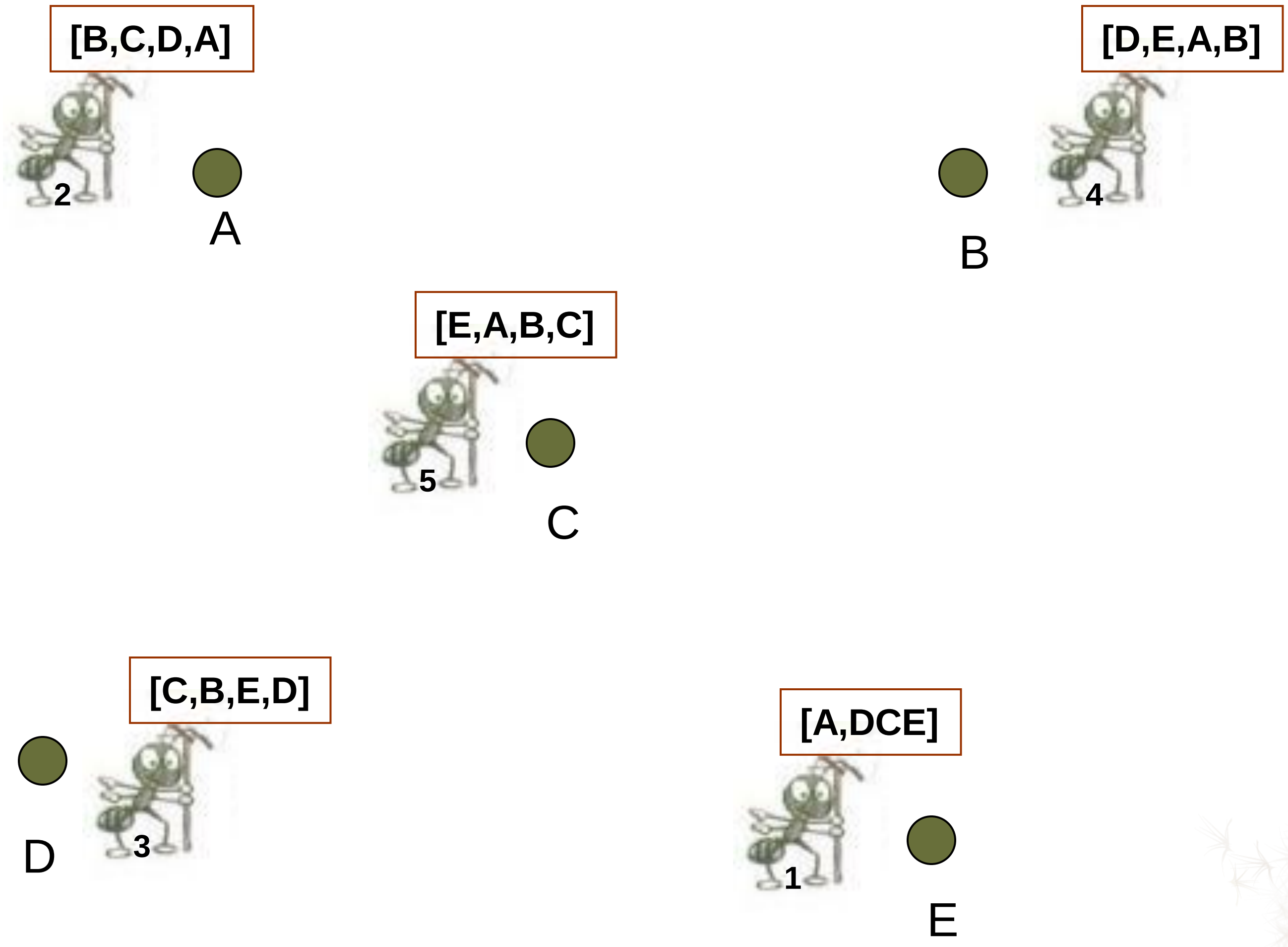
Iteration 2



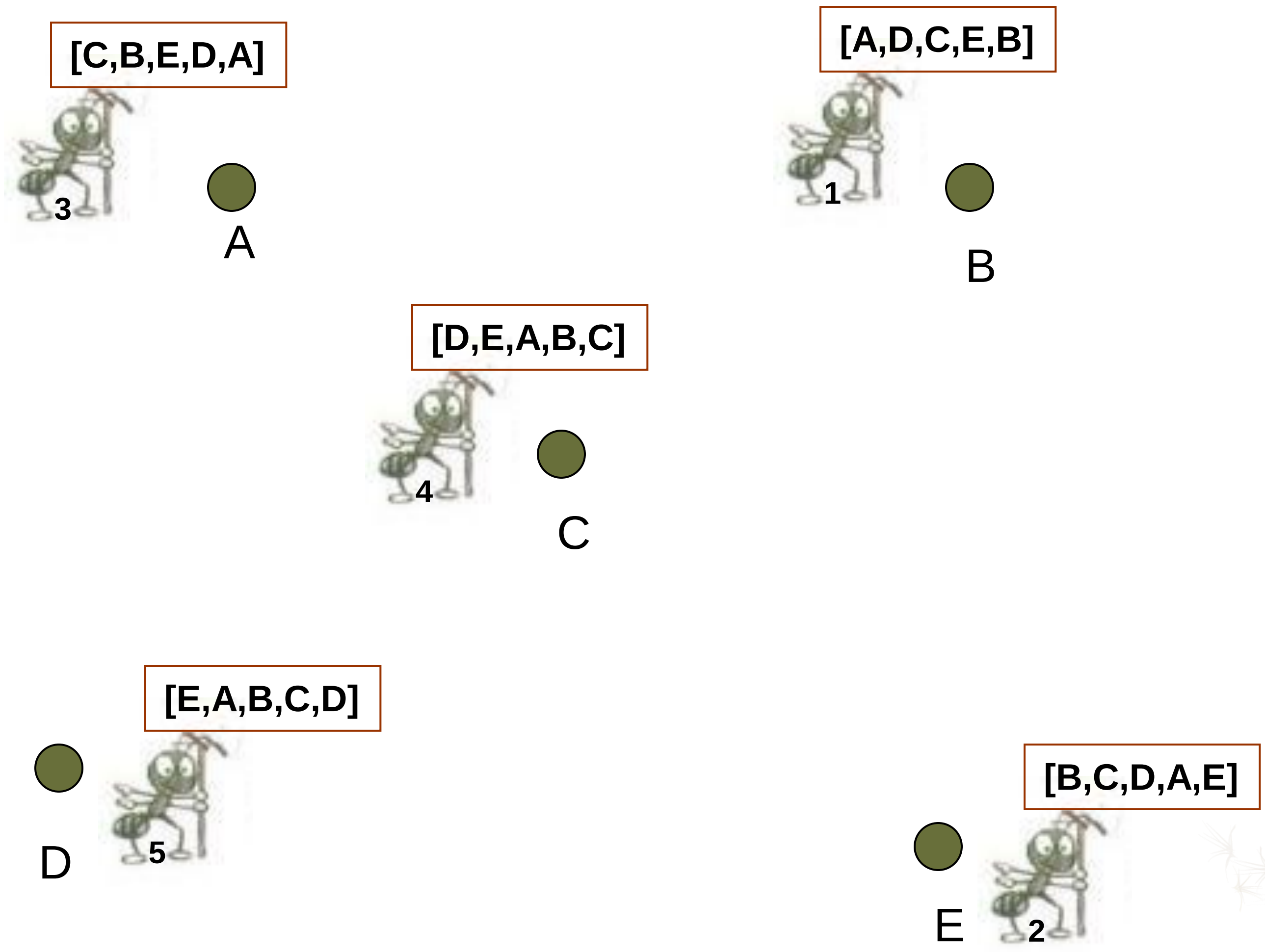
Iteration 3



Iteration 4



Iteration 5



Path and Pheromone Evaluation

[A,D,C,E,B]

L_1 =300

[B,C,D,A,E]

L_2 =450

[C,B,E,D,A]

L_3 =260

[D,E,A,B,C]

L_4 =280

[E,A,B,C,D]

L_5 =420

$$\Delta\tau_{i,j}^{k} = \begin{cases} \dfrac{Q}{L_k} & if\,(i,j) \in \text{tour} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta\tau_{A,B}^{total} = \Delta\tau_{A,B}^{1} + \Delta\tau_{A,B}^{2} + \Delta\tau_{A,B}^{3} + \Delta\tau_{A,B}^{4} + \Delta\tau_{A,B}^{5}$$

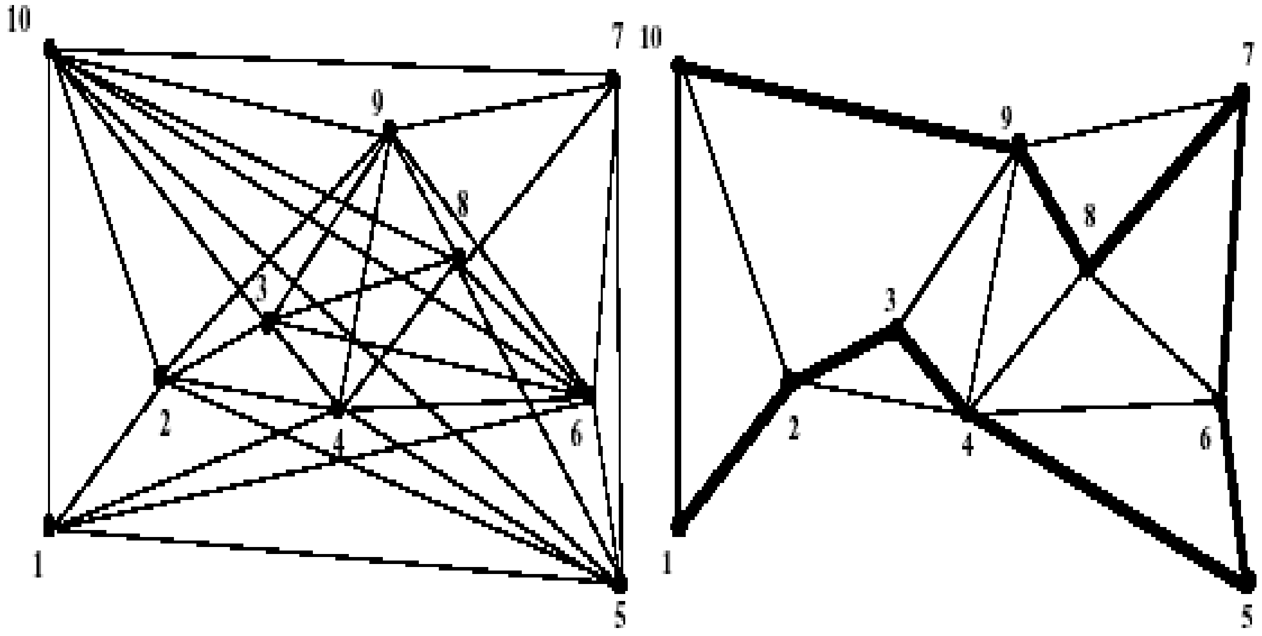
End of First Run

Save Best Tour (Sequence and length)

Do Next Run

Stopping criteria

- Stagnation (use, e.g., leaderboard)
- Max iterations



General ACO

- A stochastic construction procedure
- Probabilistically build a solution
- Iteratively add solution components to partial solutions

    - Heuristic information

    - Pheromone trail
- Reinforcement Learning reminiscence
- Modify the problem representation at each iteration

General ACO

- Ants work concurrently and independently
- Collective interaction via indirect communication leads to good solutions

Some advantages

- Positive feedback accounts for rapid discovery of good solutions
- Distributed computation avoids premature convergence
- The greedy heuristic helps find acceptable solution in the early stages of the search process.
- The collective interaction of a population of agents.

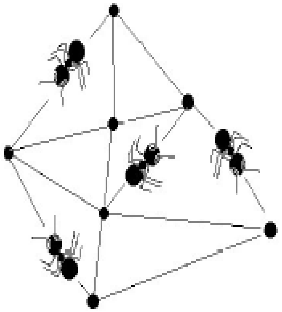
Disadvantages in Ant Systems

- Possibly slow convergence
- No centralized processor to guide the AS towards good solutions

Improvements to Ant Systems

- Also apply centralized actions
  - ACO is a local optimization procedure
  - Improve by biasing the search process with the global information
- Max-Min Ant System
  - Pheromone values are limited $\tau_{\min} \leq \tau_{ij} \leq \tau_{\max}$
  - Only the best ant(s) can add pheromones
  - Sometimes uses local search to improve its performance

Quadratic Assignment Problem(QAP)

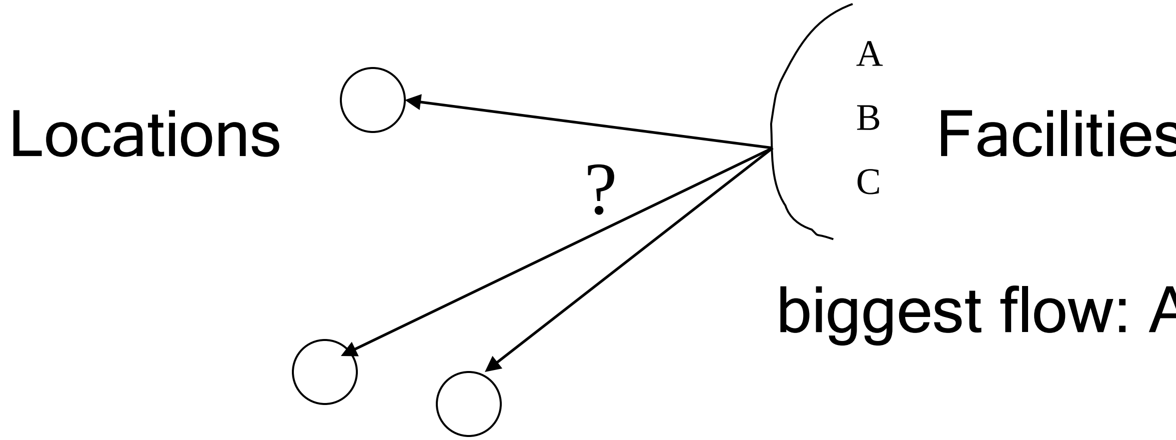


NP-hard problem defined as

- Assign n activities to n locations (campus and mall layout).
- $D = \left[d_{i,j}\right]_{n,n}$, where $d_{i,j}$ is the distance from location *i* to location *j*
- $F = \left[f_{h,k}\right]_{n,n}$, where $f_{h,k}$ is the flow from activity *h* to activity *k*
- Assignment is a permutation π
- Minimize:

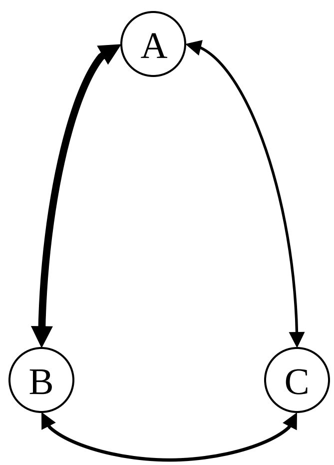
$$C(\pi) = \sum_{i,j=1}^{n} d_{ij} f_{\pi(i)\pi(j)}$$

QAP Example

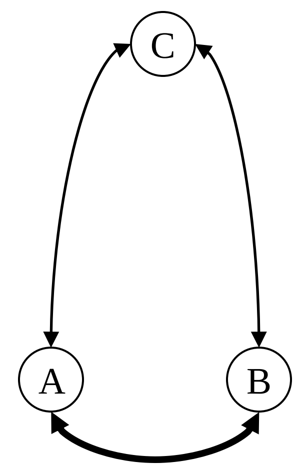


biggest flow: A - B

How to assign facilities to locations ?



Higher cost



Lower cost

SIMPLIFIED QAP

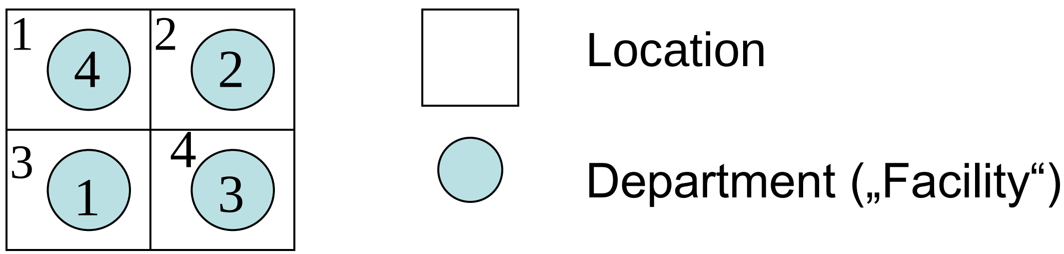
Simplification Assume all departments have equal size

Notation $d_{i,j}$ distance between **locations** i and j

$f_{k,h}$ travel frequency between **departments** k and h

$X_{i,k} \begin{cases} \text{1 if department k is assigned to location i} \\ \text{0 otherwise} \end{cases}$

Example



Distance* $d_{i,j}$

	1	2	3	4
1	-	1	1	2
2	1	-	2	1
3	1	2	-	1
4	2	1	1	-

Frequency* $f_{k,h}$

	1	2	3	4
1	-	1	3	2
2	2	-	0	1
3	1	4	-	0
4	3	1	1	-

Ant System (AS-QAP)

Constructive method:

step 1: choose a facility *j*

step 2: assign it to a location *i*

Characteristics:

– each ant leaves trace (pheromone) on the chosen couplings (i,j)

– assignment depends on the probability (function of pheromone trail and a heuristic information)

– already coupled locations and facilities are inhibited (e.g., Tabu list)

AS-QAP Heuristic information

Distance and Flow Potentials

$$D_{ij} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 4 & 5 \\ 2 & 4 & 0 & 6 \\ 3 & 5 & 6 & 0 \end{bmatrix} \Rightarrow D_i = \begin{bmatrix} 6 \\ 10 \\ 12 \\ 14 \end{bmatrix} \qquad F_{ij} = \begin{bmatrix} 0 & 60 & 50 & 10 \\ 60 & 0 & 30 & 20 \\ 50 & 30 & 0 & 50 \\ 10 & 20 & 50 & 0 \end{bmatrix} \Rightarrow F_i = \begin{bmatrix} 120 \\ 110 \\ 130 \\ 80 \end{bmatrix}$$

The coupling Matrix:

$$S = \begin{bmatrix} 720 & 1200 & 1440 & 1680 \\ 660 & 1100 & 1320 & 1540 \\ 780 & 1300 & 1560 & 1820 \\ 480 & 800 & 960 & 1120 \end{bmatrix}$$

$$s_{11} = f_1 \bullet d_1 = 720$$
$$s_{34} = f_3 \bullet d_4 = 960$$

Ants choose the location according to the heuristic desirability “Potential goodness”

$$\zeta_{ij} = \frac{1}{s_{ij}}$$

AS-QAP Constructing the Solution

- The facilities are ranked in decreasing order of the flow potentials
- Ant k assigns the facility i to location j with the probability given by:

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}(t)^{\alpha}\eta_{ij}{}^{\beta}}{\sum_{l \in N_i^k} \tau_{ij}(t)^{\alpha}\eta_{ij}{}^{\beta}} & if \quad j \in N_i^k \end{cases}$$

where N_i^k is the feasible neighborhood of node *i*

When ant k chooses to assign facility *j* to location *i,* it leaves a trace "pheromone" on the coupling *(i,j)*

- Repeated until the entire assignment is found

AS-QAP Pheromone Update

- Pheromone trail update to all couplings:

$$\tau_{ij}(t+1) = \rho\, \tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

$\Delta\tau_{ij}^{k}$ is the amount of pheromone ant k puts on the coupling *(i,j)*

$$\Delta_{ij}^{k} = \begin{cases} \dfrac{Q}{J_{\psi}^{k}} & if \;\; \text{facility i is assigned to location j in the solution of ant k} \\ 0 & \text{otherwise} \end{cases}$$

J_{ψ}^{k} ... the objective function value

Q...the amount of pheromone deposited by ant k

Hybrid Ant System For The QAP

- Constructive algorithms often result in a poor solution quality compared to local search algorithms.

- Repeating local searches from randomly generated initial solution results for most problems in a considerable gap to optimal solution

- Hybrid algorithms combining solution constructed by (artificial) ant "probabilistic constructive" with local search algorithms yield significantly improved solution.

Hybrid Ant System For The QAP (HAS-QAP)

- HAS-QAP uses of the pheromone trails in a non-standard way. It is used to modify an existing solution
- Improves the ant's solution using the local search algorithm.
- Intensification and diversification mechanisms.

Hybrid Ant System For The QAP (HAS-QAP)

```
Generate m initial solutions, each one associated to one ant
   Initialise the pheromone trail
   For Imax iterations repeat
     For each ant k = 1,..., m do
       Modify ant k;s solution using the pheromone trail
       Apply a local search to the modified solution
       new starting solution to ant k using an intensification mechanism
     End For
     Update the pheromone trail
     Apply a diversification mechanism
End For
```

HAS-QAP Intensification& diversification mechanisms

- The intensification mechanism is activated when the best solution produced by the search so far has been improved.

- The diversification mechanism is activated if during the last *S* iterations no improvement to the best generated solution is detected.

Particle Swarm Optimization (PSO)

- A population based stochastic optimization technique
- Searches for an optimal solution in the computable search space
- Developed in 1995 by Eberhart and Kennedy
- Inspired by social psychology
- Inspiration: swarms of bees, flocks of birds, schools of fish

PSO principles

- In PSO individuals strive to improve themselves and often achieve this by observing and imitating their neighbors
- Each PSO individual has the ability to remember
- PSO has simple algorithms and low overhead
  - Making it more popular in some circumstances than Genetic/Evolutionary Algorithms
  - Has only one operation calculation:
    - Velocity: a vector of numbers that are added to the position coordinates to move an individual

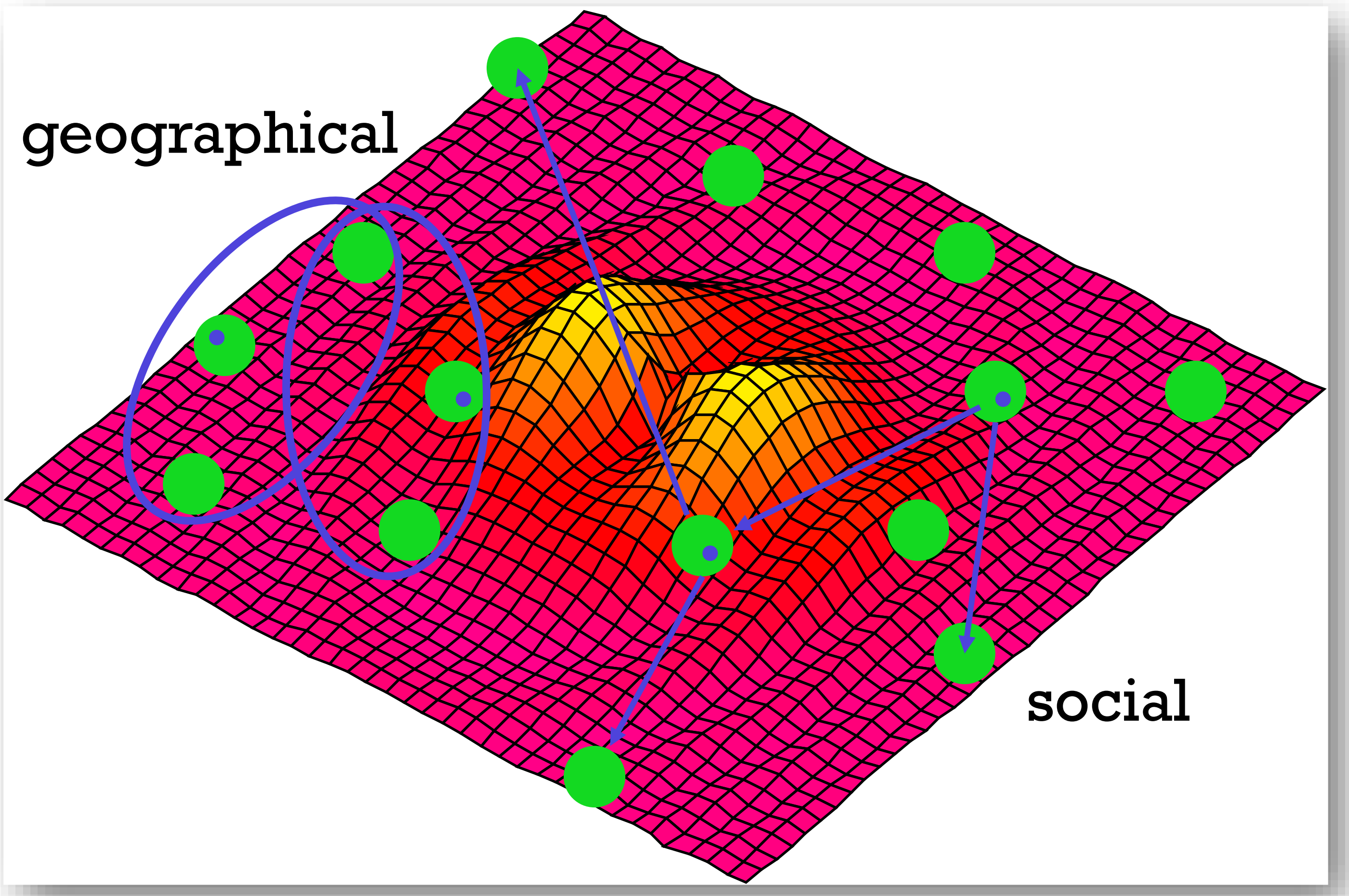
PSO and social psychology

- Individuals (points) tend to
  - Move towards each other
  - Influence each other
  - Why?
    - Individuals want to be in agreement with their neighbors
- Individuals (points) are influenced by:
  - Their previous actions/behaviors
  - The success achieved by their neighbors

What Happens in PSO

- Individuals in a population learn from previous experiences and the experiences of those around them
- The direction of movement is a function of:
  - Current position
  - Velocity (or in some models, probability)
  - Location of individuals “best” success
  - Location of neighbors “best” successes
  - Location of globally “best” success
- Therefore, each individual in a population will gradually move towards the “better” areas of the problem space
- Hence, the overall population moves towards “better” areas of the problem space

PSO: Neighborhood



Particle Swarm Optimization (PSO)

- One can imagine that each particle is represented with two vectors, location and velocity
  - Location $x = (x_1, x_2, \ldots)$
  - *Velocity* $v = (v_1, v_2, \ldots)$
  - For locations $x(t-1)$ and $x(t)$ in time t-1 and t:

$$\vec{v} = \vec{x}\,(t) - \vec{x}\,(t-1)$$

- Initialization of locations and velocities (small initial values, e.g., one half of distance to the neighboring particle, random, or 0)

Information exchange in the swarm

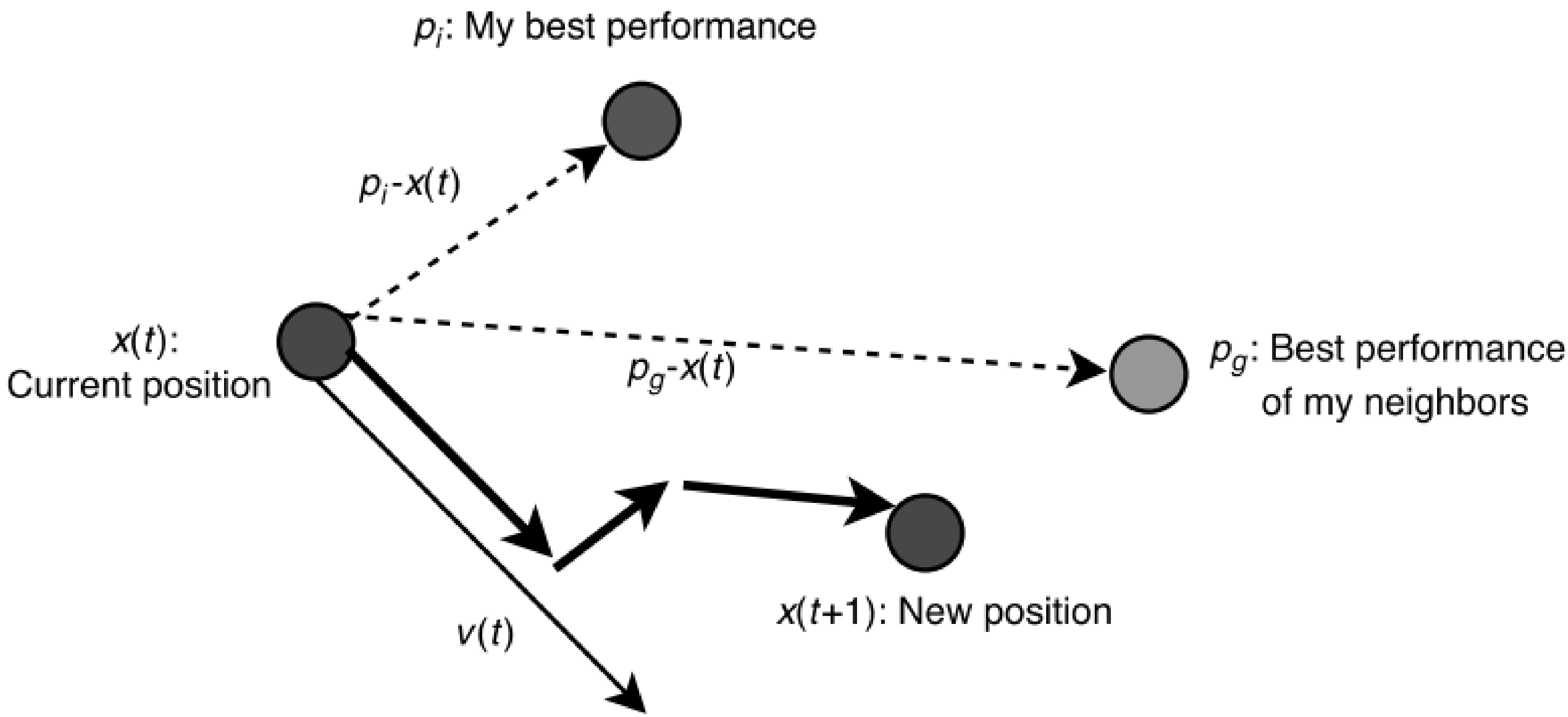
- Historically best location x^*
- Best location of informants x^+
- Globally best location $x^!$

Moving particles

- In each time step, the following operations are executed

1. compute the fitness of each particle and update x^*, x^+ in $x^!$
2. update the representation of particle
   - velocity vector takes into account updated directions x^*, x^+ in $x^!$
   - each direction is updated with some random noise
3. move the particle in the direction of velocity vector

Computing new position



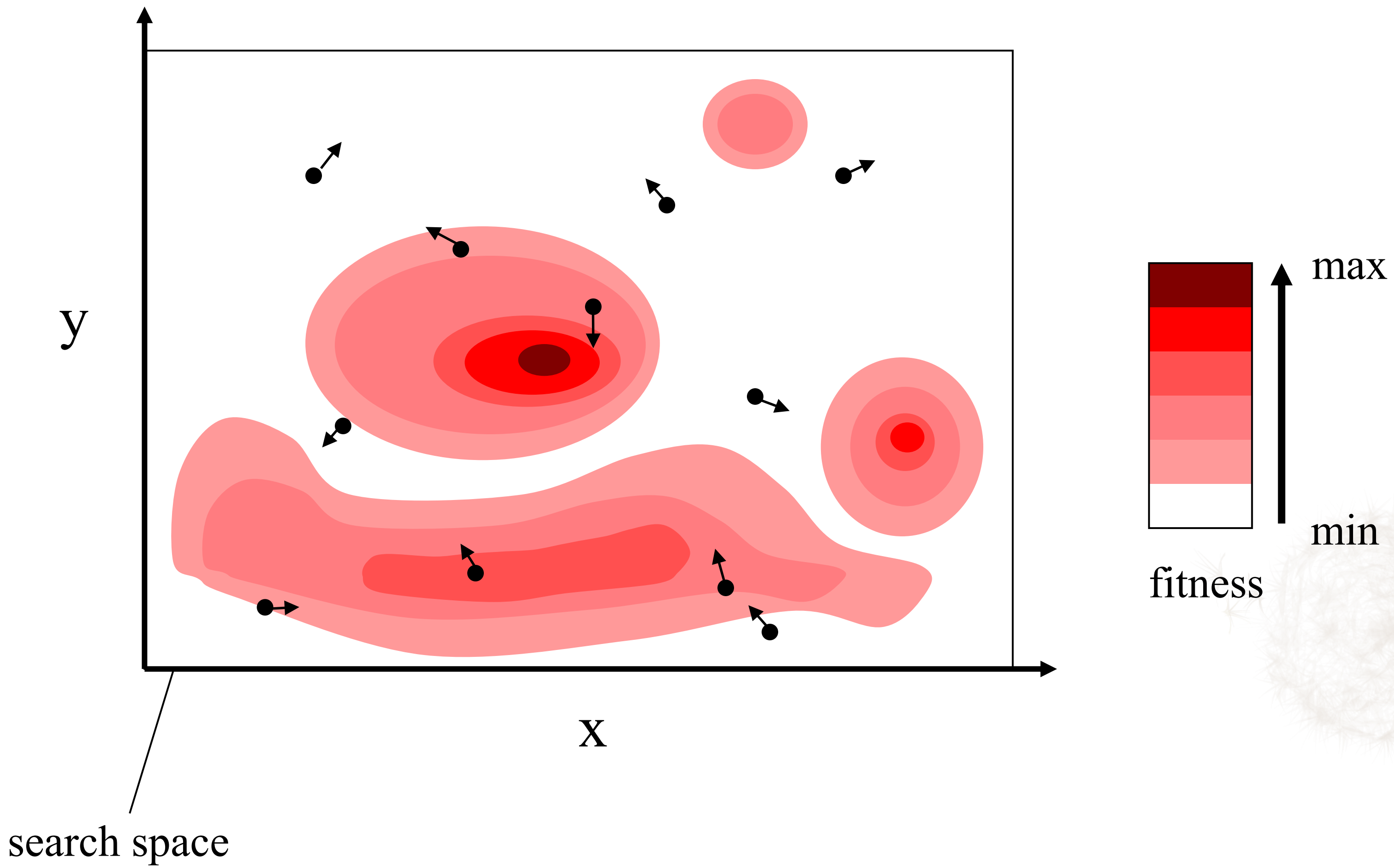
PSO - parameters

- α - proportion of current velocity vector v
- β - proportion of the best value of location x*
too large value pushes towards its maximum and we get a swarm of greedy searchers and no group dynamics
- δ - proportion of the best global location $x^!$
too large value pushes particles towards the current global maximum and we get a single greedy search, instead of several local searches (often we set this parameter to 0)
- γ - proportion of the best value of informants x^+
the effect between β and δ, depends also on the number of informants: more informants emphasize global, less informants emphasize effect of local information
- ε - speed of particle movement
too large speed may cause too fast convergence without enough search (default value is 1)
- swarmsize – size of swarm (between 20 and 50)

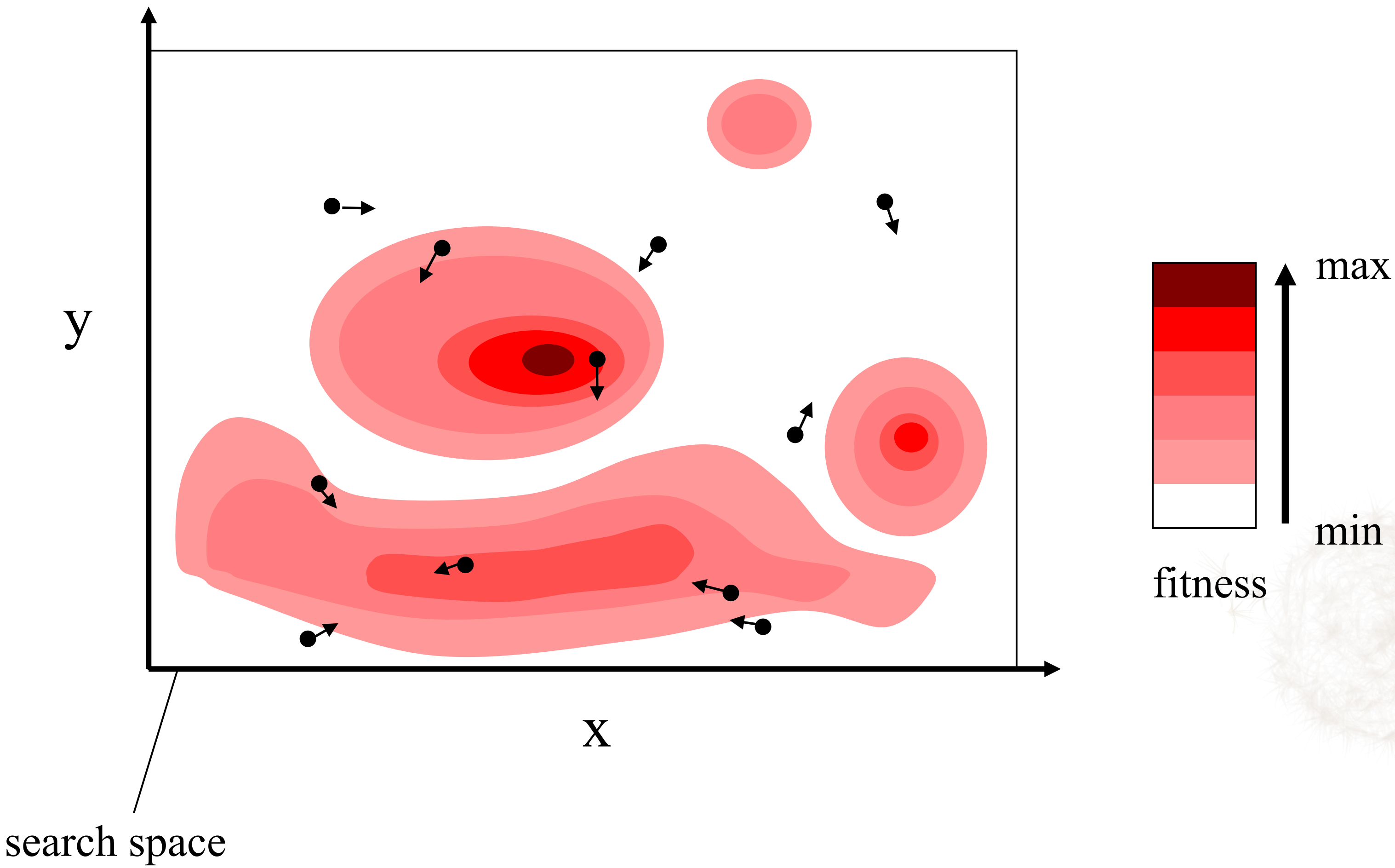
PSO pseudocode

```
P = []
for (i=0 ; i < swarmsize ; i++)
    Pi = new particle with random position x and random velocity v
best = null
do {
   for  (i=0 ; i < swarmsize ; i++) {
       compute fitness(Pi)
       if  ( fitness(Pi) >  fitness(best) )
           best = Pi
   }
   for (i=0 ; i < swarmsize ; i++)  {
      x* = update location of the best fitness of xi
      x+ = update location of the best fitness of informants of xi
      x! = update location of the best fitness of all particles
      for (j=0; j < #dimensions; j++) {
         b = random between 0 and β
         c = random between 0 and γ
         d = random between 0 and  δ
         vj = αvj + b(x*j - xj)  + c(x+j - xj) + d(x!j - xj)
      }
      xi = xi + ε·v
} while (!satisfied with best or out of time)
return best
```

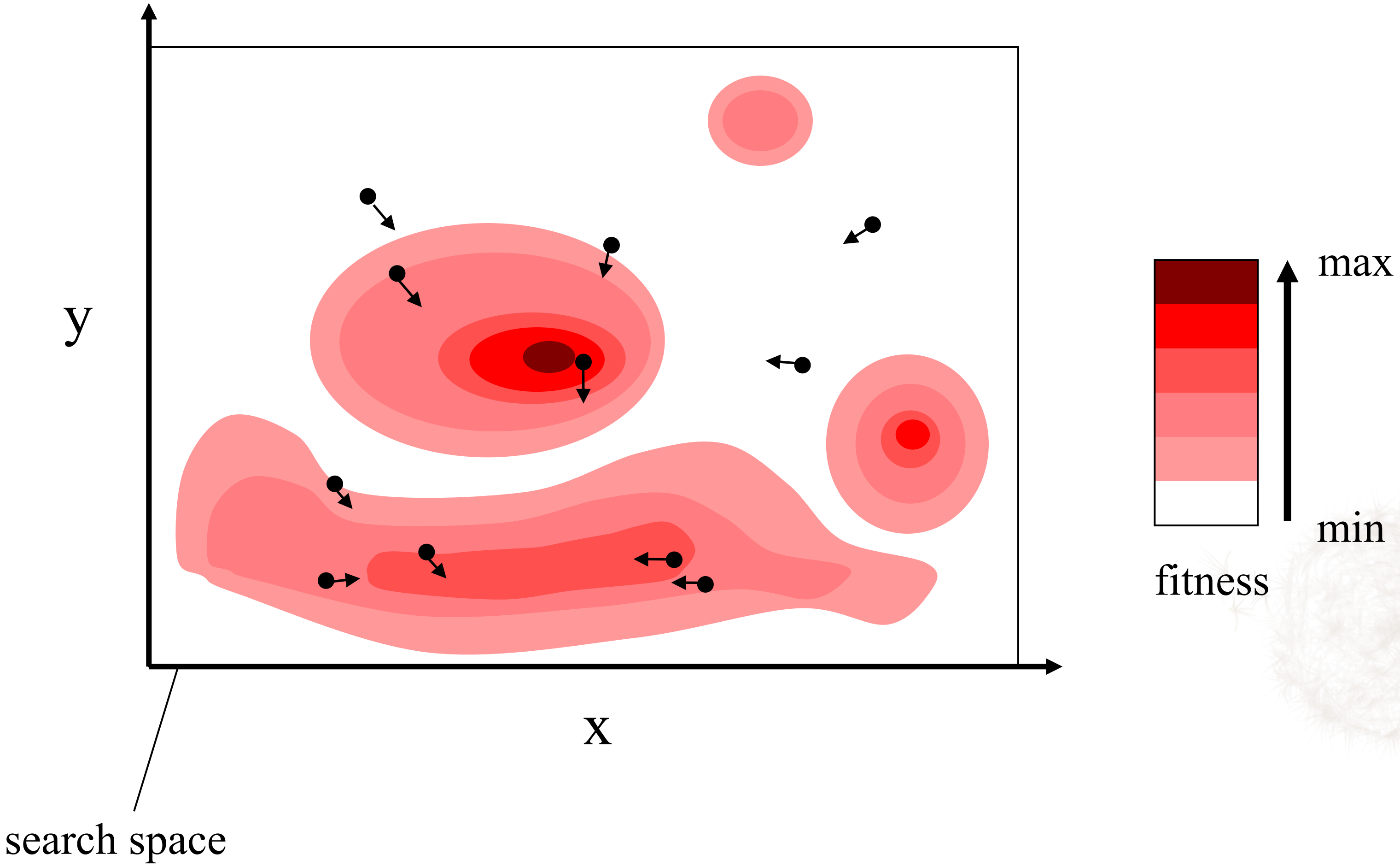
simulation 1



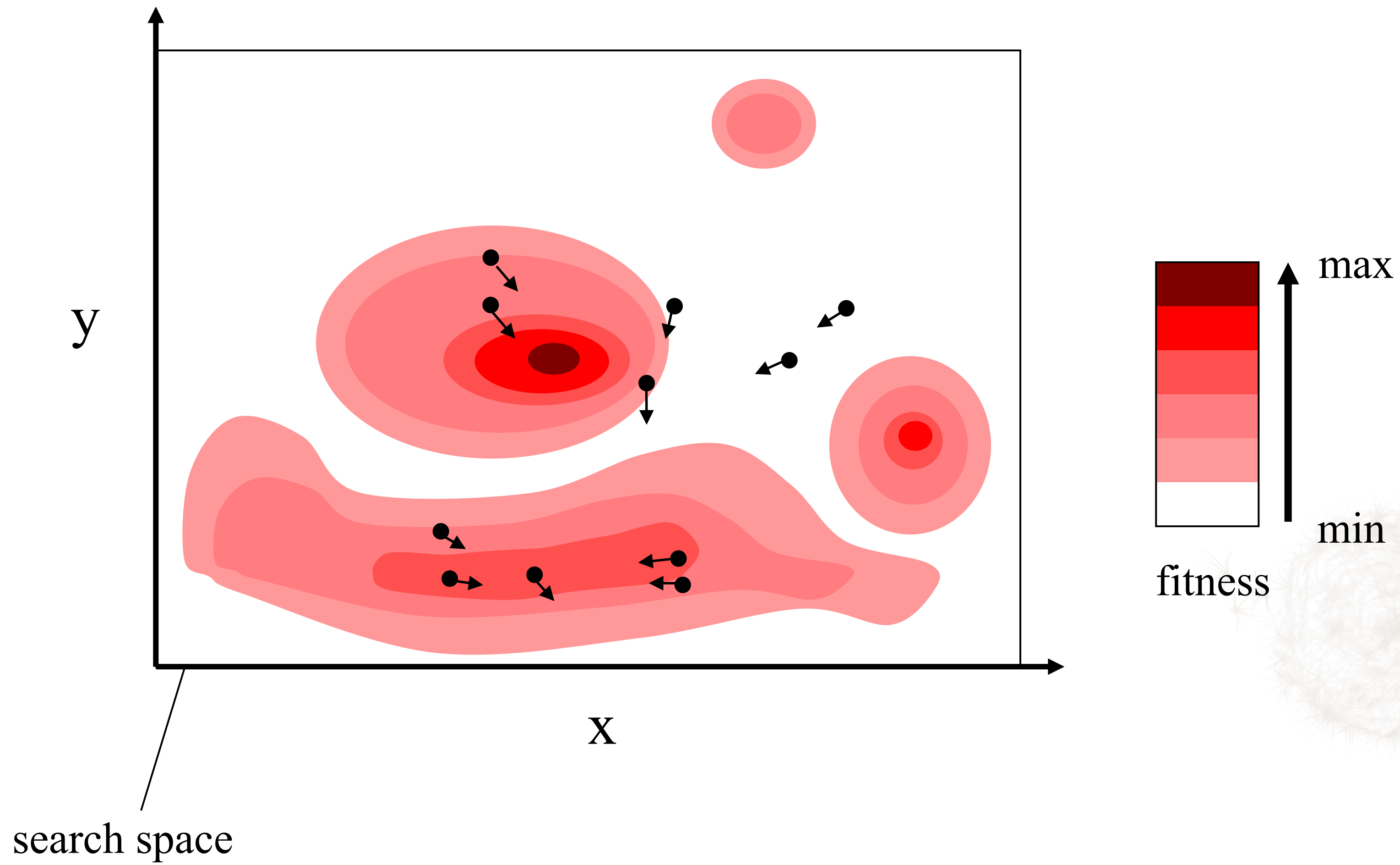
simulation $_2$



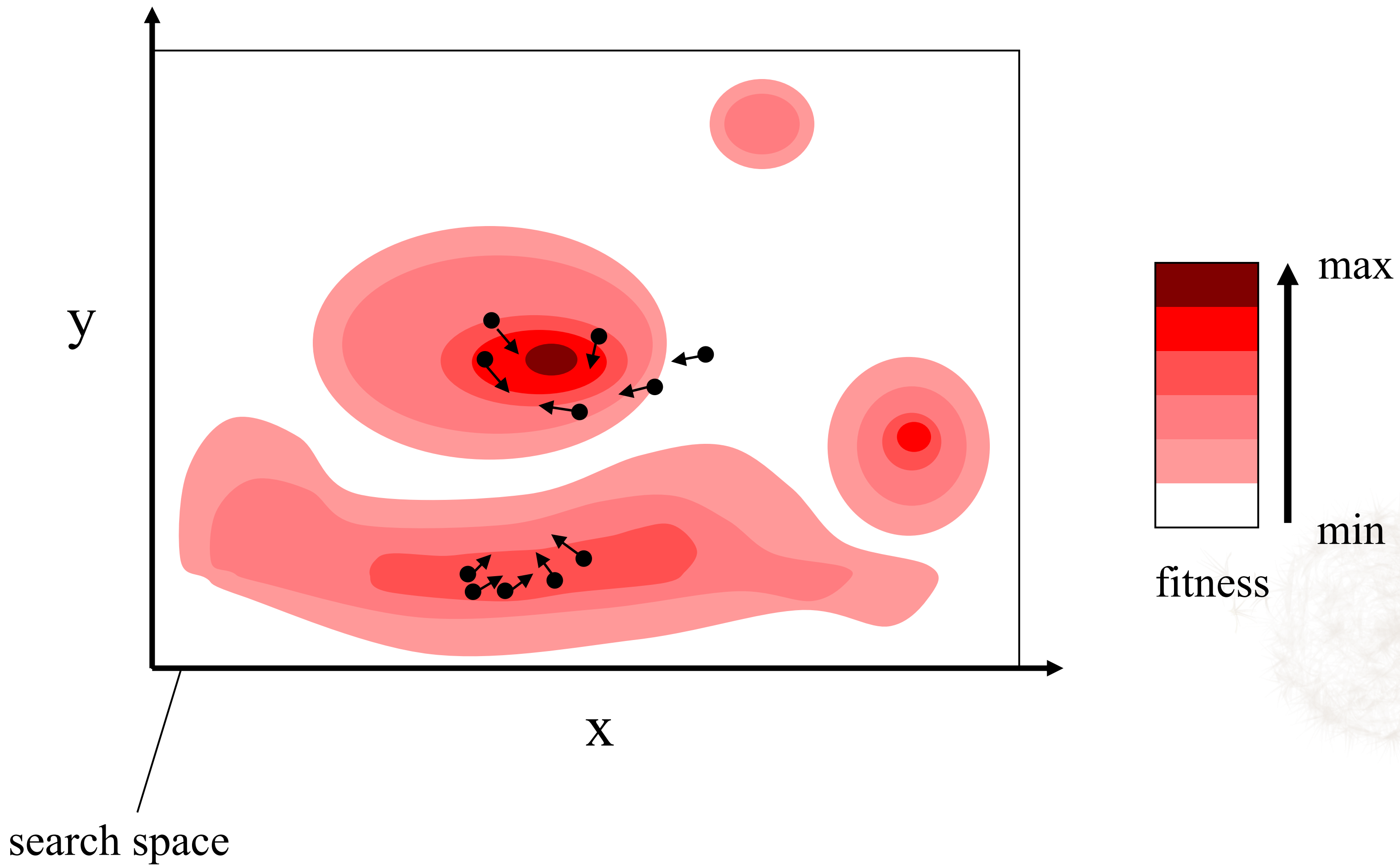
simulation [3]



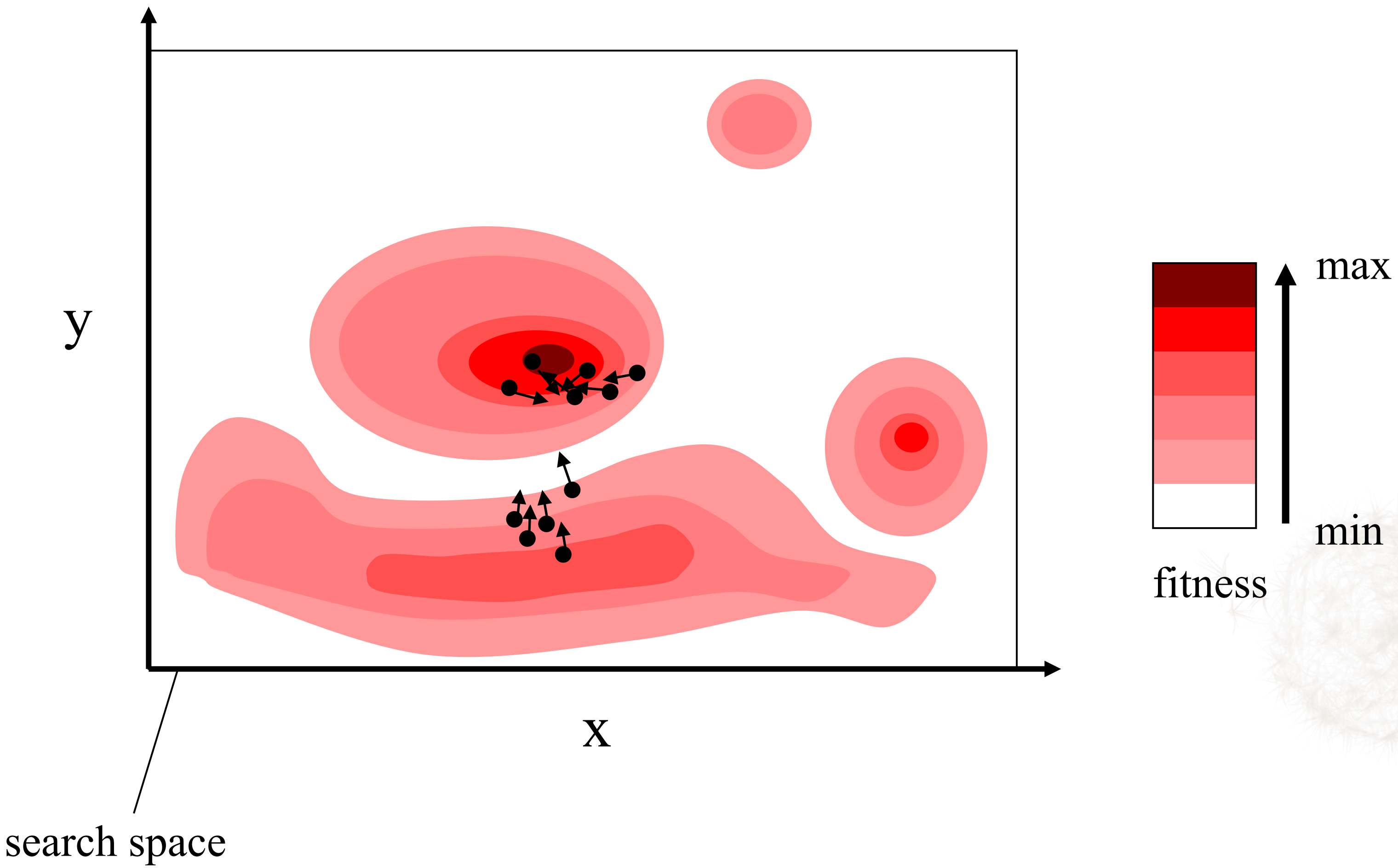
simulation 4



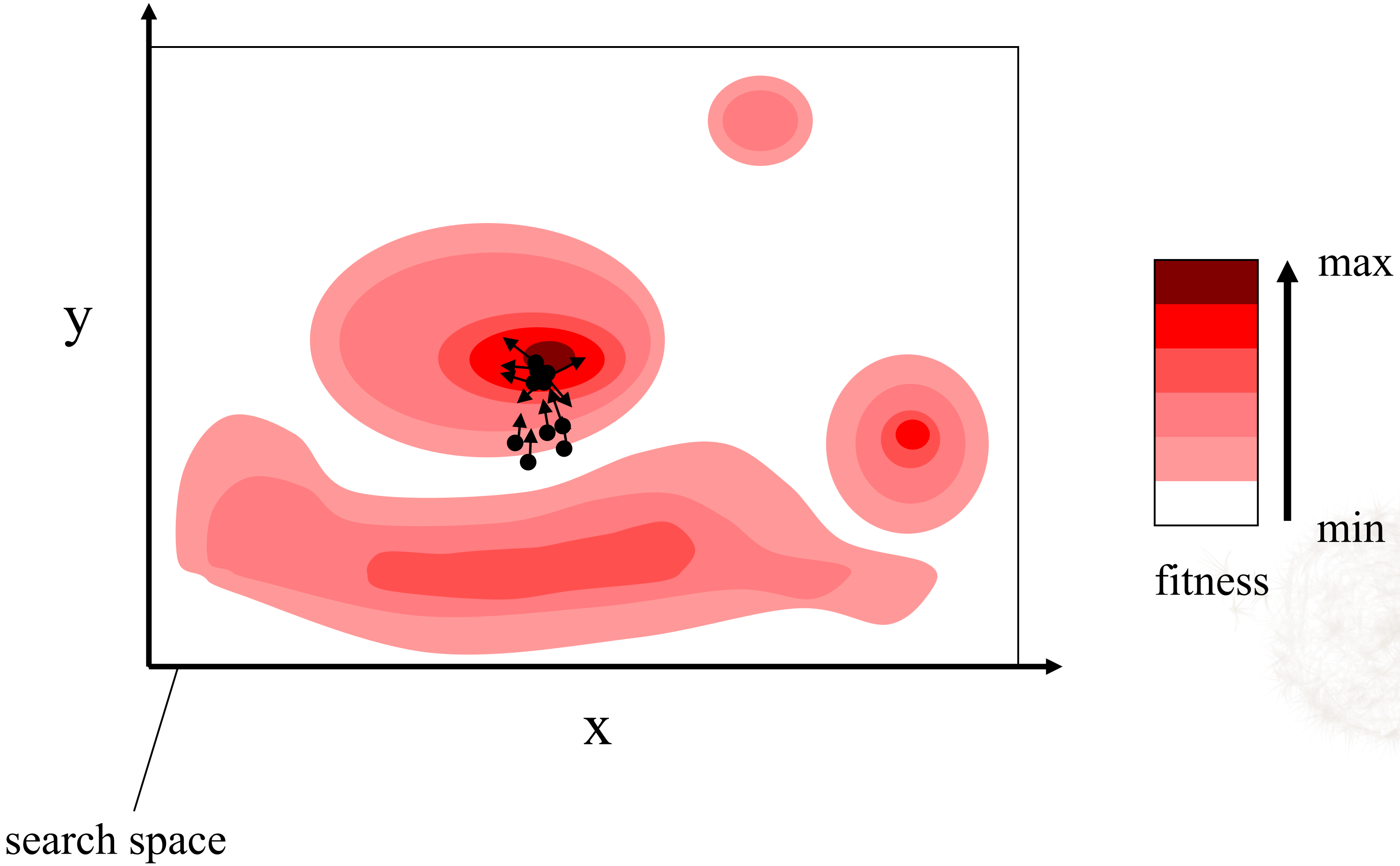
simulation 5



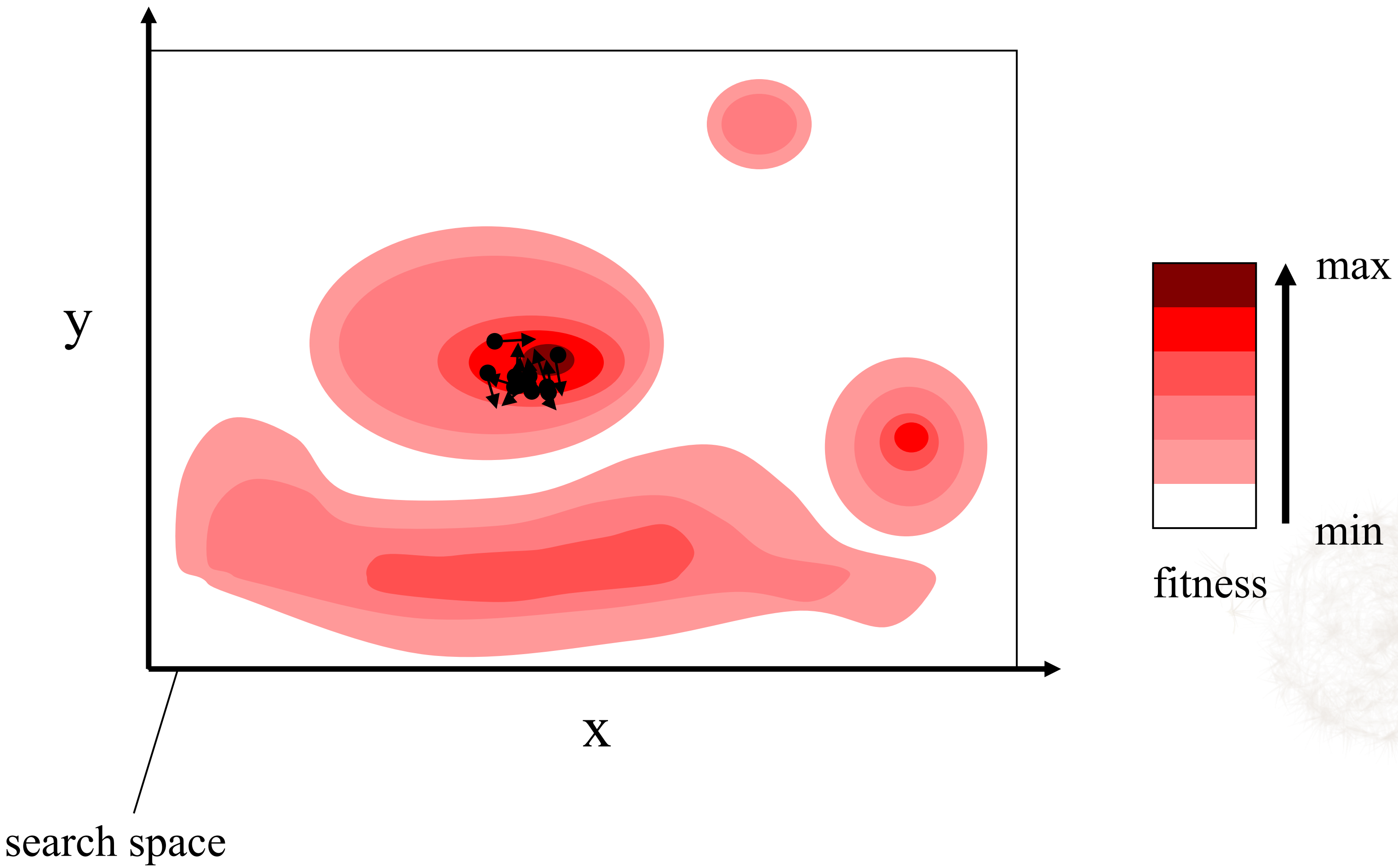
simulation 6



simulation [7]



simulation 8



PSO characteristics

- **Advantages**
  - Insensitive to scaling of design variables
  - Simple implementation
  - Easily parallelized for concurrent processing
  - Derivative free
  - Very few algorithm parameters
  - Very efficient global search algorithm
- **Disadvantages**
  - Tendency to a fast and premature convergence in mid optimum points
  - Slow convergence in refined search stage (weak local search ability)