Assignment 1

Solve the following five exercises. Each assignment is worth three points. Solutions must be submitted by 3.11.2019. Use the link on e-ucilnica to submit your solutions. Solution must be submitted in pdf format.

Exercise 1

a) Find tight asymptotic bound of the given algorithm.

```plaintext
Algorithm 1
1: function FUN(int n, int m)
2:    while n > 0 do
3:        m++;
4:        for (int i = m; i > 1; i = i / 4) do
5:            UpdateData(n, i) //θ(n)
6:        end for
7:        m--
8:        for (int i = 3m; i > m ; i/2) do
9:            UpdateData(n, 3m − i) //θ(1)
10:       end for
11:       n--
12:    end while
13:    return 0
14: end function
```

b) Find the tight asymptotic bound of the following function f(a,s,e) in relation to e-s. Hint: a - array, s - start, e - end

```plaintext
int f(int[] a, int s, int e) {
    if (s == e)
        return 7;
    int p1 = 0;
    int j = e - s;

    for(int i = j; i > 1; i-- )
        p1 += a[s + i] * a[e − i]

    int p2 = f(a, s, s + floor(j / 6));
    int p3 = f(a, s + floor(j / 6) + 1, e);
    return (p1+p2+p3);
}
```
c) Find the tight asymptotic bound of the following program in relation to variable N.

```java
Move(N, source, dest, via){
  if(N == 0) then{
    MakeMove(N, source, dest) //O(1)
  }else{
    Move(N-1, source, via, dest);
    MakeMove(N, source, dest) //O(1)
    Move(N-1, via, dest, source);
  }
}
```

**Exercise 2**

Solve the following two recurrences using Masters method.

a) 
\[ T(n) = 4 \cdot T\left(\frac{n}{9}\right) + \sqrt{n} \]  
(1)

b) 
\[ T(n) = 5 \cdot T\left(\frac{n}{4}\right) + \log(n) \]  
(2)

**Exercise 3**

Solve the following two recurrences using Akra-Bazi method.

a) 
\[ T(n) = 2T\left(\frac{n}{4}\right) + 3T\left(\frac{n}{6}\right) + \theta(n\log n) \]  
(3)

b) 
\[ T(n) = T\left(\frac{n}{9}\right) + T\left(\frac{n}{4}\right) + 2T\left(\frac{n}{36}\right) + \sqrt{n^3} \]  
(4)

**Exercise 4**

a) Estimate upper asymptotic bound of the given recurrence using recursion-tree method.

\[ T(n) = 8T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 27T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + n^3 \]  
(5)

b) Prove result in a) using substitution method. (2 points)
d) Find the tight asymptotic bound of the given recurrence. (With any method)
Exercise 5

a) Solve the following recurrence using annihilator method. Find the exact solution. (2 points)

\[ T(n) = T(n - 1) + n^2 + n - 1; T(0) = 1 \]  \hspace{1cm} (6)

b) Solve the following recurrence using annihilator method. Find the exact solution. (2 points)

\[ T(n) = 9T(n - 1) - 28T(n - 2) + 36T(n - 3) - 16T(n - 4) \]
\[ T(0) = 0; T(1) = 8 \]
\[ T(2) = 28; T(3) = 112 \]