



# Digitalna vezja UL, FRI



Vaja 2 Booleova algebra, logisim

# 1 Booleova algebra

## Operacije:

Negacija NE (NOT):

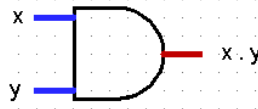
$$\bar{x} = x' = \sim x$$



x	$\bar{x}$
0	1
1	0

Konjunkcija: IN (AND)

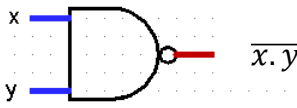
$$x \& y = x \wedge y = x \cdot y$$



x	y	$x \cdot y$	$\overline{x \cdot y}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

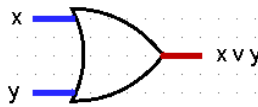
Negirana konjunkcija: NAND

$$\overline{x \cdot y} = x \uparrow y$$



Disjunkcija: OR (ALI)

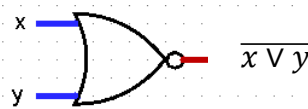
$$x \vee y = x + y$$



x	y	$x \vee y$	$\overline{x \vee y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Negirana disjunkcija: NOR

$$\overline{x \vee y} = x \downarrow y$$



## □ Zakoni

Komutativnost:

$$x \cdot y = y \cdot x$$

$$x \vee y = y \vee x$$

Distributivnost:

$$x \cdot (y \vee z) = (x \cdot y) \vee (x \cdot z) = x \cdot y \vee x \cdot z$$

$$x \vee (y \cdot z) = (x \vee y) \cdot (x \vee z)$$

Konstanta (0, 1):

$$x \vee 0 = x$$

$$x \cdot 1 = x$$

Komplement ( $x, \bar{x}$ )

$$x \cdot \bar{x} = 0$$

$$x \vee \bar{x} = 1$$

Asociativnost - izrek

$$(x \cdot y) \cdot z = x \cdot (y \cdot z) = x \cdot y \cdot z$$

$$(x \vee y) \vee z = x \vee (y \vee z) = x \vee y \vee z$$

Konstanta (0, 1) - izrek

$$x \cdot 0 = 0$$

$$x \vee 1 = 1$$

Idempotenca - izrek

$$x \cdot x = x$$

$$x \vee x = x$$

Vsebovanost - izrek

$$x \cdot (x \vee y) = x$$

$$x \vee (x \cdot y) = x$$

Dvojna negacija:

$$\bar{\bar{x}} = x$$

DeMorganov izrek

$$\overline{x \cdot y} = \bar{x} \vee \bar{y}$$

$$\overline{x \vee y} = \bar{x} \cdot \bar{y}$$

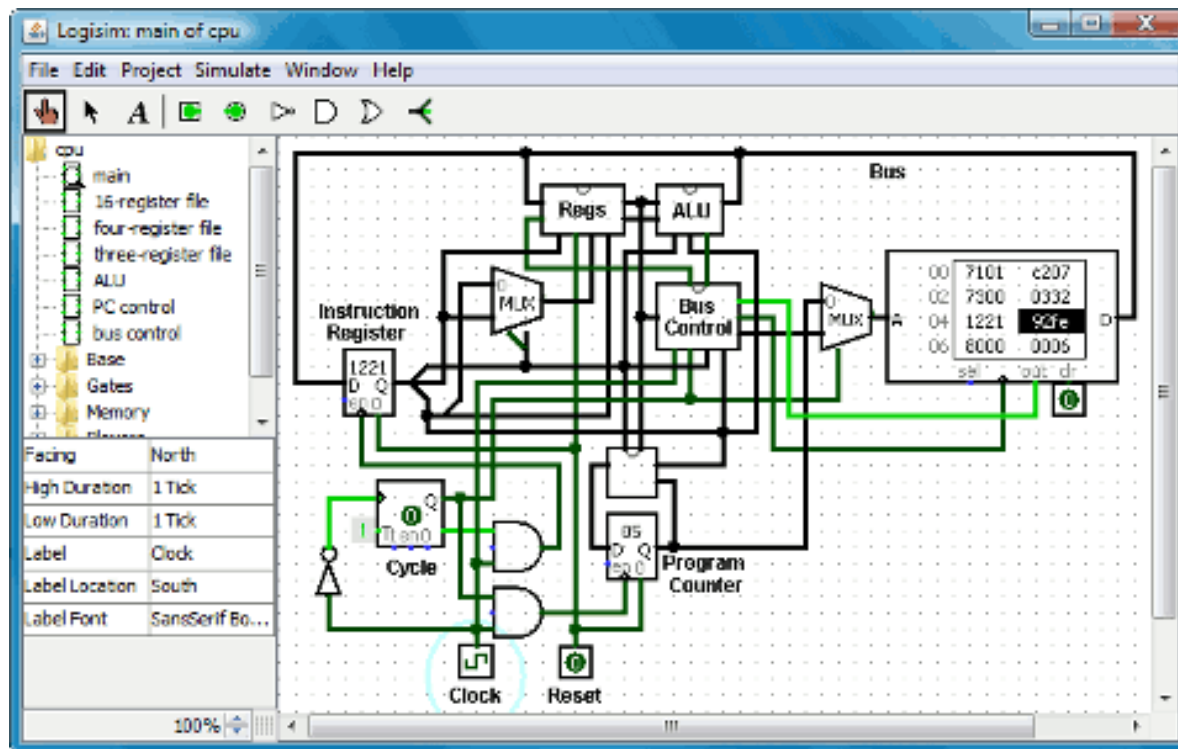
$$\overline{x \cdot y \cdot z} = \bar{x} \vee \bar{y} \vee \bar{z}$$

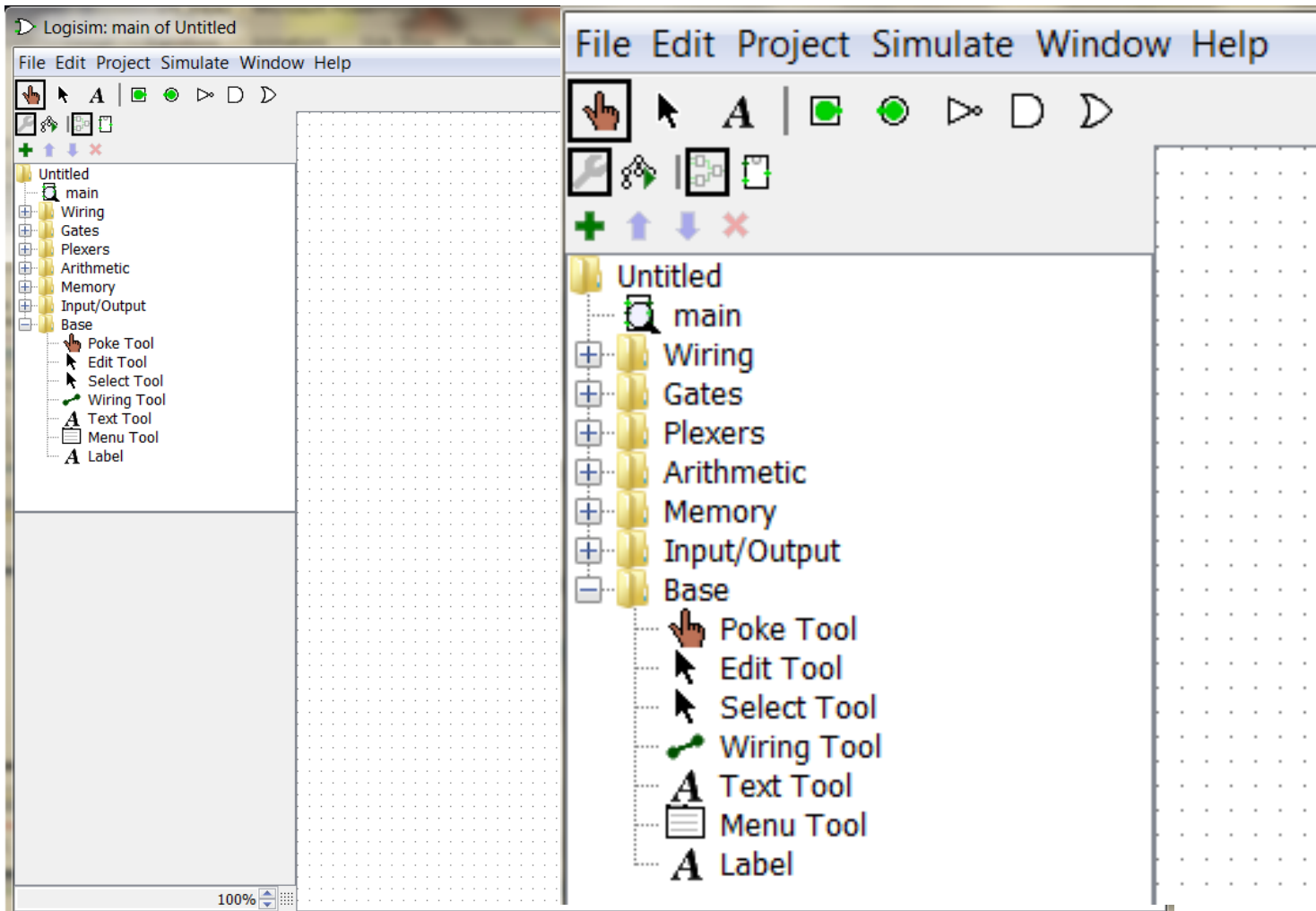
$$\overline{x \vee y \vee z} = \bar{x} \cdot \bar{y} \cdot \bar{z}$$

## 2 Logisim

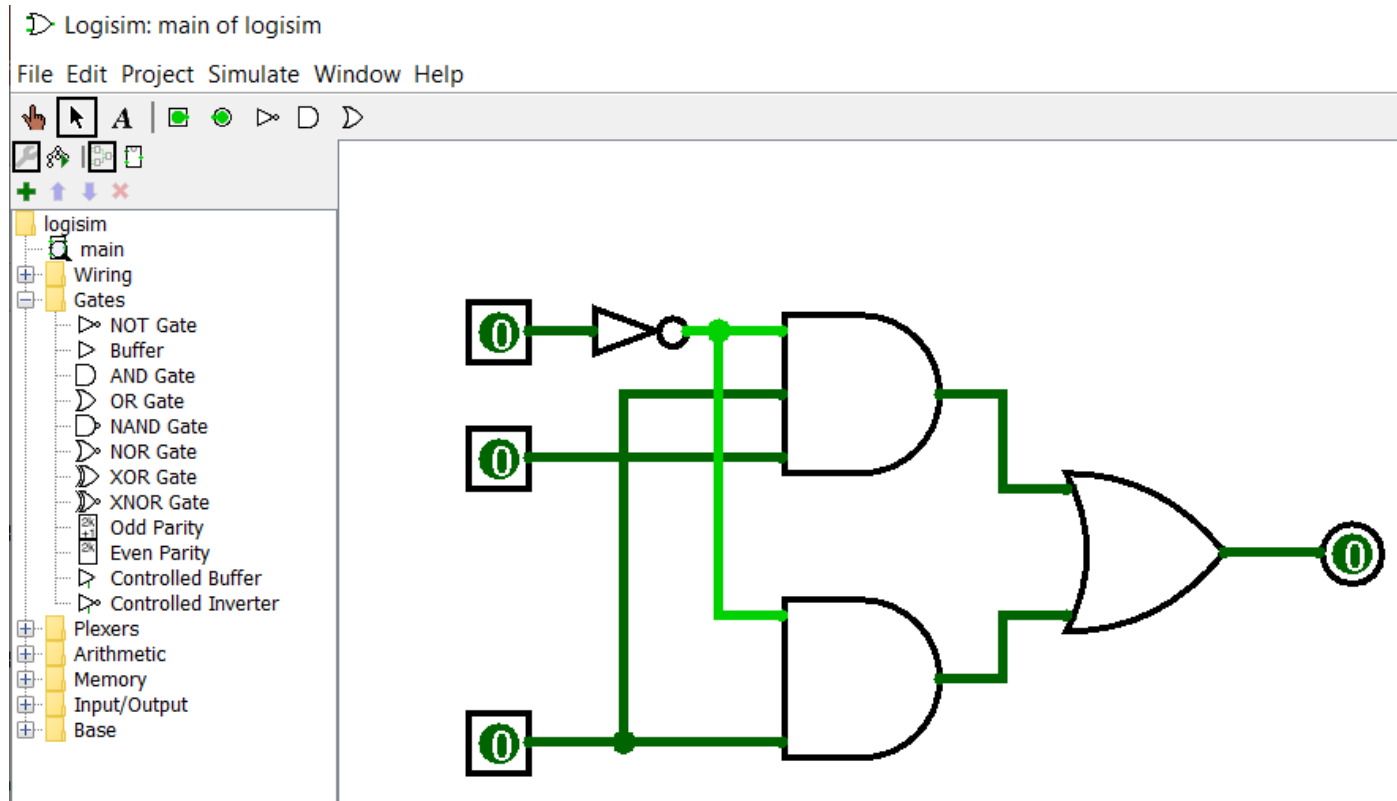
- Načrtovanje in simulacija digitalnih/logičnih vezij (<http://www.cburch.com/logisim/index.html>)

logisim-win-2.7.1.exe



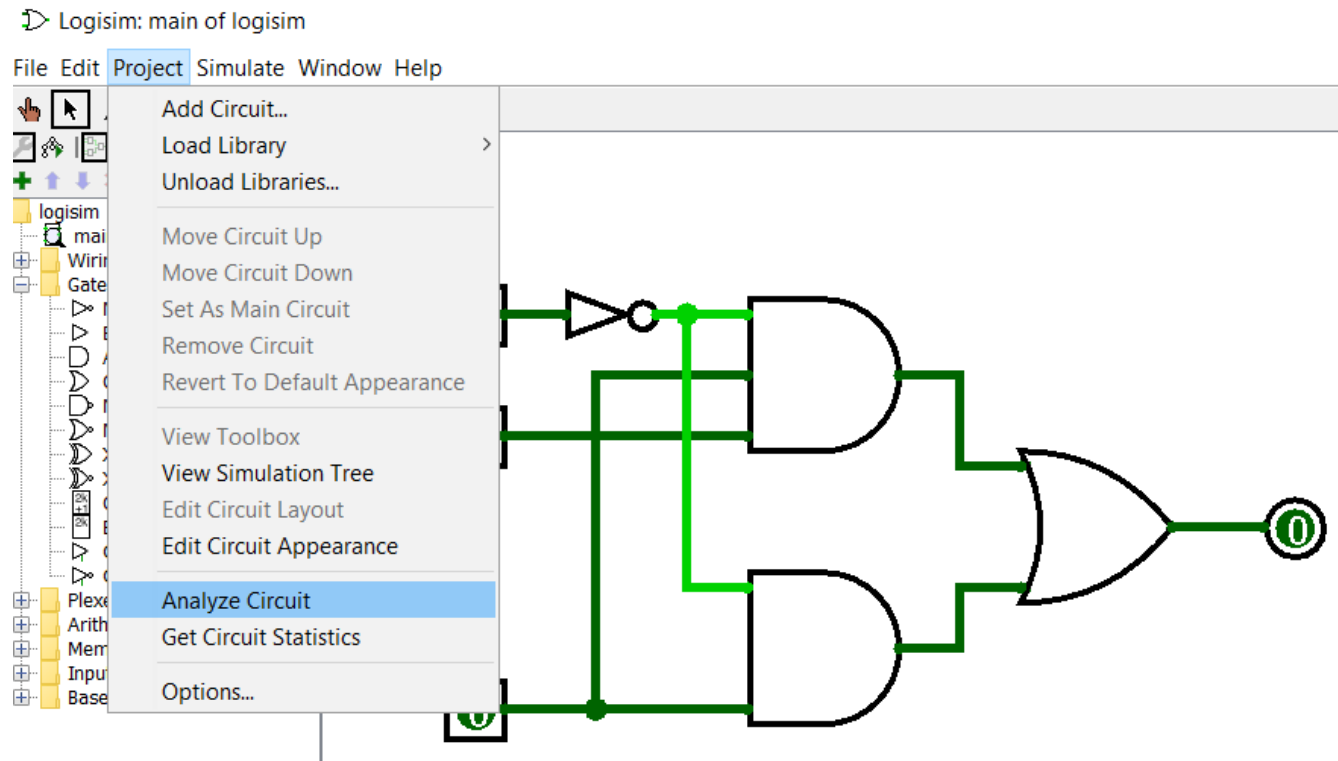


- Narišite vezje in preverite izhod za vse možne vhodne kombinacije s spreminjanjem vhodnih vrednosti

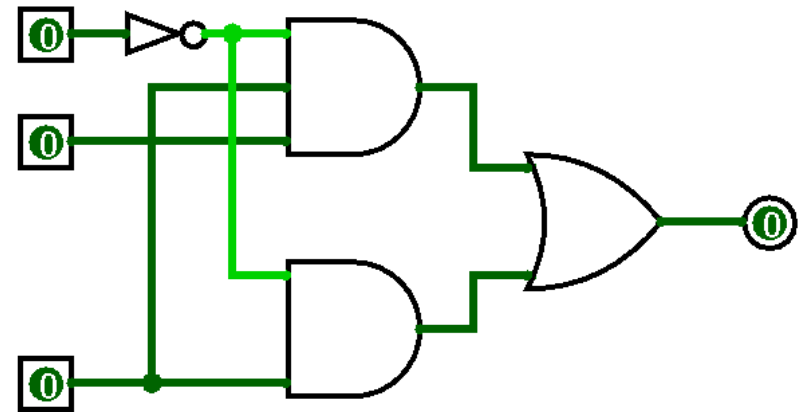


## ❑ Analiza vezja (ang. Analyse Circuit):

- Vhodi in izhodi
- Tabela
- Zapis funkcije
- Minimizacija



- Vhodi (a,b,c) in izhod (x) – simulator določi oznake vhodov in izhodov
- Tabela
- Zapis funkcije
- Minimizacija



Combinational Analysis

File Edit Project Simulate Window Help

Inputs Outputs **Table** Expression Minimized

View and manipulate the truth table.

a	b	c	x
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Build Circuit

Combinational Analysis

File Edit Project Simulate Window Help

Inputs Outputs Table **Expression** Minimized

Output: x

$\bar{a}c b + \bar{a}c$

$\sim a c b + \sim a c$

Clear Revert Enter

Build Circuit

Combinational Analysis

File Edit Project Simulate Window Help

Inputs Outputs Table Expression **Minimized**

Output: x

Format: Sum of products

		b, c			
		00	01	11	10
a	0	0	1	1	0
	1	0	0	0	0

$\bar{a}c$

Set As Expression

Build Circuit



# Naloga 1: Poenostavljanje logičnih funkcij

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□ Poenostavljanje logičnih funkcij z uporabo Booleove algebre

□ Primer:

$$f(x, y, z) = \bar{x} \cdot \bar{y} \cdot z \vee \bar{x} \cdot y \cdot z = \bar{x} \cdot z \cdot (\bar{y} \vee y) = \bar{x} \cdot z \cdot 1 = \bar{x} \cdot z$$

$$\begin{aligned} f(x, y, z) &= \bar{x} \cdot \bar{y} \cdot z \vee x \cdot \bar{y} \cdot \bar{z} \vee x \cdot \bar{y} \cdot z \vee \bar{x} \cdot y \cdot z = \\ &= \bar{x} \cdot z (\bar{y} \vee y) \vee x \cdot \bar{y} \cdot (\bar{z} \vee z) \\ &= \bar{x} \cdot z \vee x \cdot \bar{y} \end{aligned}$$

□ Poenostavite logične funkcije.

1.  $f(x, y, z) = \bar{x} \cdot \bar{y} \cdot z \vee x \cdot y \cdot z \vee x \cdot y \cdot \bar{z} \vee x \cdot \bar{y} \cdot z = ?$

2.  $f(x, y, z, u) = \bar{x} \cdot \bar{y} \cdot z \cdot u \vee x \cdot y \cdot z \cdot u \vee x \cdot y \cdot \bar{z} \cdot u \vee x \cdot y \cdot z \cdot \bar{u} = ?$

3.  $f(x, y) = \overline{\bar{x} \cdot \bar{y}} \vee x \cdot y = ?$

4.  $f(x, y, z) = \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z)} \vee (x \vee z) = ?$

5.  $f(A, B, C, D) = A \cdot C \vee \overline{(\bar{A} \cdot \bar{B} \vee A \cdot B)} \cdot (\bar{C} \vee \overline{A \cdot D}) = ?$

6.  $f(A, B, C, D) = A \vee C \cdot \overline{(\bar{A} \cdot \bar{B} \vee B)} \vee \bar{C} \cdot \overline{\bar{B} \vee \bar{D}} = ?$

# Naloga 1 Rešitve

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$$\begin{aligned} 1) f(x, y, z) &= \bar{x} \cdot \bar{y} \cdot z \vee x \cdot y \cdot z \vee x \cdot y \cdot \bar{z} \vee x \cdot \bar{y} \cdot z = \\ &= \bar{y} \cdot z \cdot (\bar{x} \vee x) \vee x \cdot y \cdot (\bar{z} \vee z) = \bar{y} \cdot z \vee x \cdot y \end{aligned}$$

$$\begin{aligned} 2) f(x, y, z) &= \bar{x} \cdot \bar{y} \cdot z \cdot u \vee x \cdot y \cdot z \cdot u \vee x \cdot y \cdot \bar{z} \cdot u \vee x \cdot y \cdot z \cdot \bar{u} = \\ &= z \cdot u \cdot (\bar{x} \cdot \bar{y} \vee x \cdot y) \vee x \cdot y \cdot (\bar{z} \cdot u \vee z \cdot \bar{u}) \end{aligned} \quad \text{ali}$$

$$f(x, y, z) = \bar{x} \cdot \bar{y} \cdot z \cdot u \vee x \cdot y \cdot u (z \vee \bar{z}) \vee x \cdot y \cdot z \cdot \bar{u} = \bar{x} \cdot \bar{y} \cdot z \cdot u \vee x \cdot y \cdot u \vee x \cdot y \cdot z \cdot \bar{u}$$

$$\begin{aligned} 3) f(x, y, z) &= \overline{(\bar{x} \cdot \bar{y} \vee x \cdot y)} = \bar{x} \cdot \bar{y} \cdot \bar{x} \cdot \bar{y} = (x \vee y) \cdot (\bar{x} \vee \bar{y}) = \\ &= x \cdot \bar{x} \vee x \cdot \bar{y} \vee \bar{x} \cdot y \vee y \cdot \bar{y} = x \cdot \bar{y} \vee \bar{x} \cdot y \end{aligned}$$

$$\begin{aligned} 4) f(x, y, z) &= \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z) \vee (x \vee z)} = \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z)} \cdot \overline{x \vee z} = \\ &= (\bar{x} \cdot \bar{y}) \cdot (\bar{y} \cdot \bar{z}) \cdot (\bar{x} \cdot \bar{y}) = (x \vee y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{x} \cdot \bar{z} = \\ &= (x \cdot \bar{y} \vee x \cdot \bar{z} \vee y \cdot \bar{y} \vee y \cdot \bar{z}) \cdot \bar{x} \cdot \bar{z} = \bar{x} \cdot y \cdot \bar{z} \end{aligned} \quad \text{ali}$$

$$\begin{aligned} f(x, y, z) &= \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z) \vee (x \vee z)} = \bar{x} \cdot \bar{y} \vee y \cdot z \vee x \vee z = \\ &= \bar{x} \cdot \bar{y} \cdot \bar{y} \cdot \bar{z} \cdot \bar{x} \cdot \bar{z} = (x \vee y) \cdot (\bar{y} \vee \bar{z}) \cdot \bar{x} \cdot \bar{z} = \\ &= (x \cdot \bar{y} \vee x \cdot \bar{z} \vee y \cdot \bar{y} \vee y \cdot \bar{z}) \cdot \bar{x} \cdot \bar{z} = \bar{x} \cdot y \cdot \bar{z} \end{aligned}$$

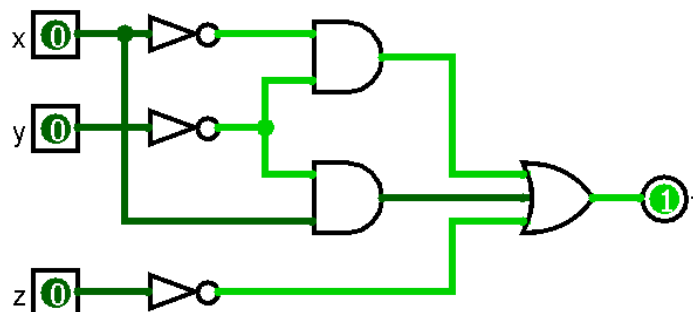
$$\begin{aligned}
f(A, B, C, D) &= A.C \vee \overline{(\bar{A}.\bar{B} \vee A.B)}.(\bar{C} \vee \overline{A.D}) = \\
&= A.C \vee \overline{\bar{A}.\bar{B}.\bar{A}.\bar{B}}.(\bar{C} \vee \bar{A} \vee \bar{D}) = \\
&= A.C \vee (A \vee B).(\bar{A} \vee \bar{B}).(\bar{C} \vee \bar{A} \vee \bar{D}) = \\
&= A.C \vee (A.\bar{B} \vee \bar{A}.B).(\bar{C} \vee \bar{A} \vee \bar{D}) = \\
&= A.C \vee (A.\bar{B}.\bar{C} \vee A.\bar{B}.\bar{D} \vee \bar{A}.B.\bar{C} \vee \bar{A}.B.\bar{D}) =
\end{aligned}$$

$$\begin{aligned}
f(A, B, C, D) &= A \vee C. \overline{(\bar{A}.\bar{B} \vee B)} \vee \bar{C}.\overline{B \vee \bar{D}} = \\
&= A \vee C.\bar{A}.\bar{B}.\bar{B} \vee \bar{C}.(\bar{B}.D) = \\
&= A \vee C.(A \vee B).\bar{B} \vee \bar{C}.\bar{B}.D = \\
&= A \vee C.A.\bar{B} \vee \bar{C}.\bar{B}.D = \\
&= C.\bar{B} \vee \bar{C}.\bar{B}.D
\end{aligned}$$

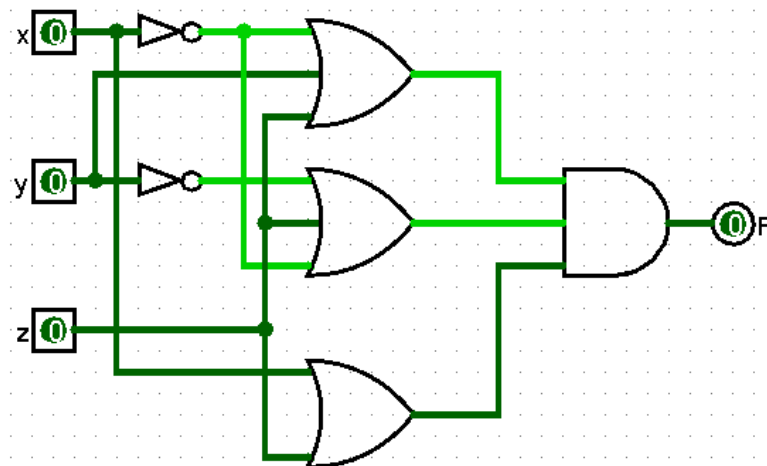
## Naloga 2

- Zapišite izhod logičnega vezja (f in F) v algebraičnem zapisu z operatorji NOT, AND, OR

1)  $f = ?$



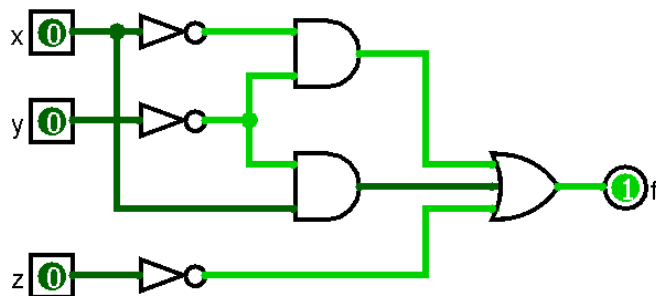
2)  $F = ?$



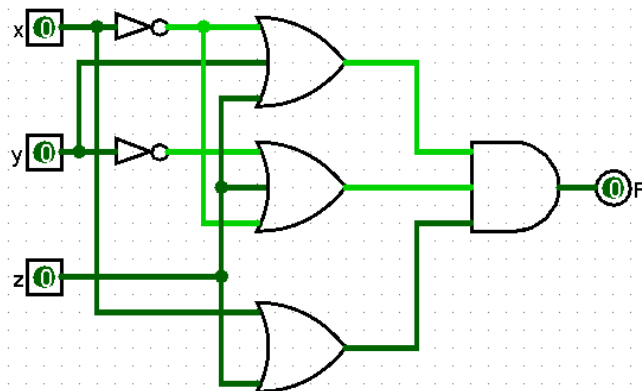
## Naloga 2 Rešitev

Algebraični zapis logičnih funkcij z operatorji NOT, AND, OR

$$1) f = \bar{x} \cdot \bar{y} \vee x \cdot \bar{y} \vee \bar{z}$$



$$F = (\bar{x} \vee y \vee z) \cdot (\bar{x} \vee \bar{y} \vee z) \cdot (x \vee z)$$



## Naloga 3

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1. V podano pravilnostno tabelo zapišite logično funkcijo (varianta a) ali b):

$$f(A, B, C) = A \cdot \bar{B} \vee \bar{A} \cdot \bar{B} \vee A \cdot B \cdot C$$

1. Narišite vezje v logisimu in preverite rezultat.

a)

A	B	C	$A \cdot \bar{B}$	$\bar{A} \cdot \bar{B}$	$A \cdot B \cdot C$	$f(A, B, C)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

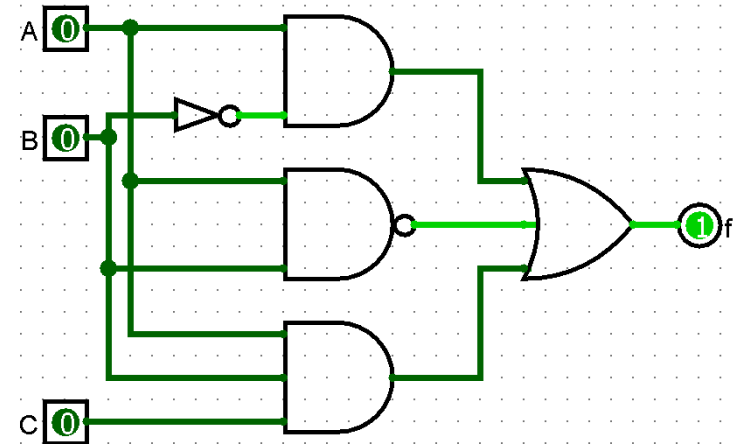
b)

A	B	C	$f(A, B, C)$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

# Naloga 3 Rešitev

$$f = A \cdot \bar{B} \vee \overline{A \cdot B} \vee A \cdot B \cdot C$$

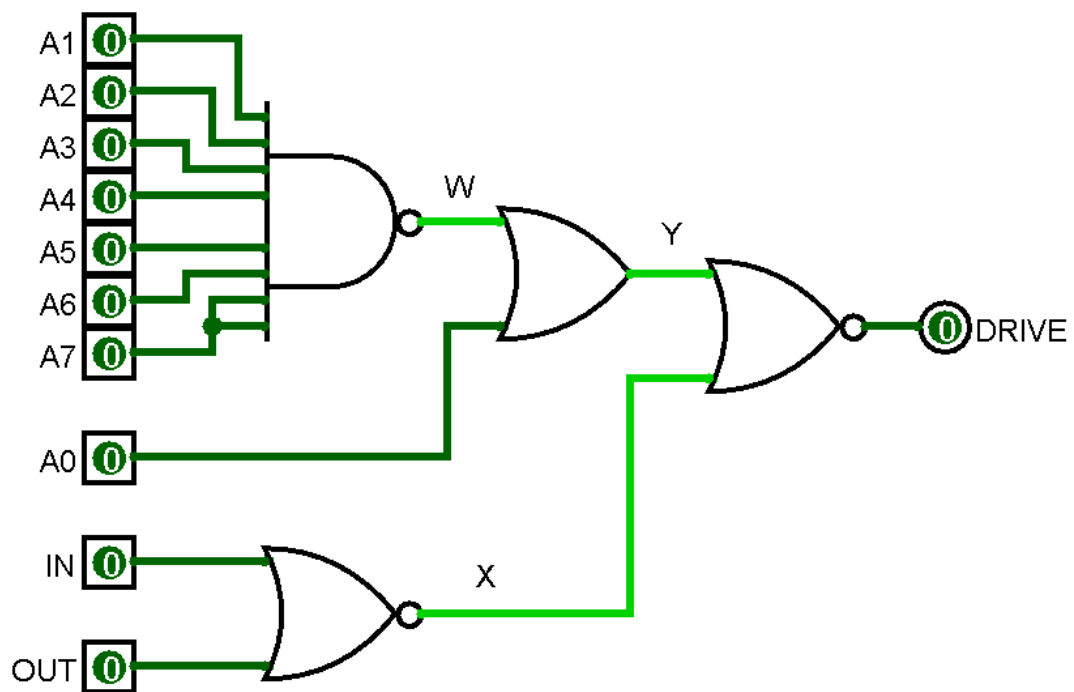
A	B	C	$A \cdot \bar{B}$	$\overline{A \cdot B}$	$A \cdot B \cdot C$	$f(A, B, C)$
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	0	0	0	0
1	1	1	0	0	1	1



A	B	C	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

## Naloga 4

Podano logično vezje se uporablja za nadzor motorja pogonskega vretena za pogon diskete, ko mikroračunalnik pošilja podatke ali jih prejema z diskete. Vezje bo vklopilo motor, ko bo signal DRIVE enak 1. Določite vhodne pogoje, ki so potrebni za vklop motorja tako, da interpretirate vrednosti signalov Y, X in W.



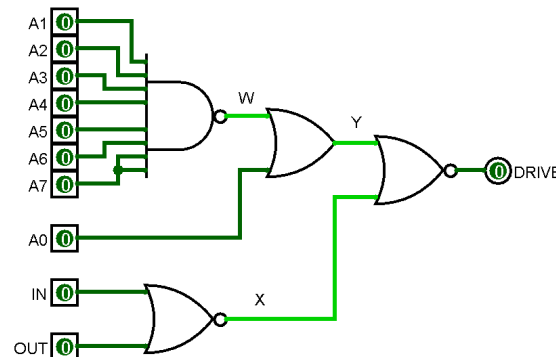


## Naloga 4 Rešitev

Podano logično vezje se uporablja za nadzor motorja pogonskega vretena za pogon diskete, ko mikroračunalnik pošilja podatke ali jih prejema z diska. Vezje bo vklopilo motor, ko bo signal DRIVE enak 1. Določite vhodne pogoje, ki so potrebni za vklop motorja. Tako, da interpretirate signale Y, X in W.

Shemo razložimo postopoma:

1. DRIVE je aktiven-HIGH in bo HIGH le, ko bo  $X = Y = 0$ .
2. X bo LOW, če je ali IN ali OUT HIGH.
3. Y bo LOW le, če sta W logična 0 in A0 logična 0.
4. W bo LOW le, če so vsi signali A1 do A7 HIGH.
5. Če vse skupaj združimo, bo DRIVE HIGH,
  - ko bodo  $A1=A2=A3=A4=A5=A6=A7=1$  in
  - $A0=0$ , bodisi IN ali OUT ali oba sta 1.



Upoštevajte simbol za osem-vhodna vrata NAND;

Upoštevajte tudi, da je signal A7 povezan z dvema vhodoma NAND.

# Domača naloga 1

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Logične funkcije zapišite z novim naborom operatorjev. Uporabite zakone in izreke Booleove algebre. Rezultat preverite v logisimu tako, da realizirate logično vezje in preverite enakost v tabeli.

Rešitve zapišite na list A4 in fotografijo ali sken oddajte na učilnici.

1) Pretvorba v zapis z operatorji AND, OR in negiranimi spremenljivkami:

$$f(A, B, C) = \overline{A \cdot \overline{B}} \vee \overline{A \cdot B} \vee \overline{A \cdot B \cdot C}$$

$$f(A, B, C, D) = \overline{\overline{A \vee \overline{B}} \vee (\overline{B \vee C})} \cdot D$$

2) Pretvorba v zapis z operatorji NAND

$$f(A, B, C) = A \vee B \cdot \overline{C}; \quad f(A, B, C) = A \cdot C \vee \overline{B} \cdot \overline{C}$$

3) Pretvorba v zapis z operatorji NOR

$$f(A, B, C) = (A \vee C) \cdot \overline{B}; \quad f(A, B, C) = (A \vee \overline{B}) \cdot (B \vee \overline{C})$$

4) Poenostavitev logične funkcije z uporabo Booleove algebre

$$f(A, B, C, D) = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} \vee \overline{A} \cdot B \cdot \overline{C} \cdot \overline{D} \vee \overline{A} \cdot B \cdot C \cdot \overline{D} \vee A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} \vee \\ A \cdot B \cdot \overline{C} \cdot \overline{D} \vee A \cdot \overline{B} \cdot \overline{C} \cdot D \vee A \cdot B \cdot C \cdot \overline{D} \vee \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D$$