Algorithms 2018/2019

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Exercises 3- E3
Homeworks/Assignments

In the course there are several requirements a student has to fulfill:

1. seminar 1: presentation of a conference; present conference on Algorithms and abstracts of three papers from the conference,
2. seminar 2: essay paper presented at the course conference
3. assignments: there will be four assignments at the course
Homeworks/Assignments

Each of the seminars is marked separately and a student has to obtain at least 40% points at each one.

Further, from the assignments student has to get at least 20% at each assignment and at least 50% on the average.

When computing the average, each assignment weights the same.

The final course mark is a weighted sum of the above three marks,
▶ the seminar 1 is weighted with 20%,
▶ seminar 2 with 50%,
▶ the average of assignments with 30%,
sum and then divided by 10 and rounded up.
Delaunay Triangulation
Voronoi diagram
Where to use Delaunay Triangulation?

Approximation of the terrain.
The height of the terrain is only measured on some predefined points, so it needs to be approximated on all the others. It is done triangulating the points.

1. But in which way?
2. Which triangulation among the all is the best one?
3. What are its geometrical properties, which we can specify i.e. to a computer and not to bother about the others?
Delaunay Triangulation

1. The skinniness in terrain approx. triangulation is bad. In other words, the height is defined by the closest measured points (vertices) of the triangulation.

2. The triangulation that contains small angles is bad. Therefore we want to maximise minimal edge and get the angle-optimal triangulation: legal triangulation.
Theorem 3.1
(9.8) Let $P$ be a set of points in the plane. A triangulation $T$ of $P$ is legal if and only if $T$ is a Delaunay triangulation of $P$.

1. Remember that any angle-optimal triangulation must be legal, thus Theorem 9.8 says that any angle-optimal triangulation is a Delaunay triangulation.

2. If $P$ is in general position, there is only one legal triangulation which implies that there is only one angle-optimal triangulation, namely the unique Delaunay triangulation (graph - then are the same when $P$ is in general position) of $P$. 
Delaunay Triangulation - Terrain

When a set of points is in general position (which is almost always), we have the following statement.

**Statement 3.1**
Delaunay triangulation is the triangulation to be used in terrain approximation problem.
Delaunay Triangulation - pseudo code I

INPUT: set of \( P \) \( n + 1 \) points in the plane

OUTPUT: A Delaunay triangulation of \( P \), \( T \)

Find lexicographically highest point \( p_0 \),

2: Let \( p_{-1} \) and \( p_{-2} \) be enough far away so that \( P \) is contained in triangle \( p_0, p_{-1}, p_{-2} \),

Init \( T \) with \( p_0, p_{-1}, p_{-2} \),

4: Compute random permutation \( p_1, p_2, \ldots, p_n \) of points \( P \setminus \{p_0\} \)

for \( r \neq n \) do

6: * Insert \( p_r \) into \( T \):

Find a triangle that includes \( p_r \)

8: if \( p_r \in \triangle_{p_i,p_j,p_k} \) then

Connect \( p_r \) with every vertex \( p_i, p_j, p_k \),

10: LEGALISEEDGE\((p_r, p_ip_j, T)\)

LEGALISEEDGE\((p_r, p_jp_k, T)\)

12: LEGALISEEDGE\((p_r, p_kp_i, T)\)

end if
Delaunay Triangulation - pseudo code II

14: \textbf{if} \quad pr \in \square pi, pl, pj, pk, \quad \text{on } \overline{pipj} \quad \textbf{then}
\begin{align*}
\text{LEGALISEEDGE}(pr, \overline{pipl}, \mathcal{T}) \\
\text{LEGALISEEDGE}(pr, \overline{pipj}, \mathcal{T}) \\
\text{LEGALISEEDGE}(pr, \overline{pjpk}, \mathcal{T}) \\
\text{LEGALISEEDGE}(pr, \overline{pkpi}, \mathcal{T})
\end{align*}
\textbf{end if}

16: \quad \text{LEGALISEEDGE}(pr, \overline{pipl}, \mathcal{T})
18: \quad \text{LEGALISEEDGE}(pr, \overline{pjpk}, \mathcal{T})

20: \quad r+ = 1

22: \quad \text{Discard } p_{-1}, p_{-2}
\quad \text{return } \mathcal{T}
Theorem 3.2
(9.6) Let $P$ be a set of points in the plane.

1. Three points $p_i, p_j, p_k \in P$ are vertices of the same face of the Delaunay graph of $P$ if and only if the circle through $p_i, p_j, p_k$ contains no point of $P$ in its interior.

2. Two points $p_i, p_j \in P$ form an edge of the Delaunay graph of $P$ if and only if there is a closed disc $C$ that contains $p_i$ and $p_j$ on its boundary and does not contain any other point of $P$.

Theorem 3.3
Let $P$ be a set of points in the plane, and let $T$ be a triangulation of $P$. Then $T$ is a Delaunay triangulation of $P$ if and only if the circumcircle of any triangle of $T$ does not contain a point of $P$ in its interior.
$p_r$ lies in the interior of a triangle

$p_r$ falls on an edge
Quad trees of set of points creating its mesh