Assignment 2

Solve the following three exercises. Each exercise is worth five points. Solutions must be submitted by 28.4.2024. Use the link on e-ucilnica to turn in your work. The submission must be in pdf format.

Drop sort

What is the expected number of non zero elements in array out[] at the end of the function given below? Assume that array[] only has positive, non zero values. Solve using indicator random variables?

Algorithm 1 Drop sort

```
1:  function DropSort(int array[])
2:      ascending = true
3:      if array[0] > array[1] then
4:          ascending = false
5:      end if
6:      int[] out = new int[array.length] //Initializes all values to 0
7:      out[0] = array[0]
8:      out[1] = array[1]
9:      int c = 1
10:     for i = 2; i < array.length(); i++ do
11:         if ascending then
12:             if array[i] ≥ out[c] then
13:                 c++
14:                 out[c] = array[i]
15:             end if
16:         else
17:             if array[i] < out[c] then
18:                 c++
19:                 out[c] = array[i]
20:             end if
21:         end if
22:     end for
23:     return out
24: end function
```

Bloom filter

A Bloom filter is a data structure for checking if an element is member of a set. It has two functions add(element) and check(element). Function add(element) remembers each added element. Function check(element) returns $TRUE$ if the element is in the data structure and $FALSE$ otherwise. If the check function returns $FALSE$ the element is not in the Bloom filter. If it returns $TRUE$ there is a small probability that the element is not in the structure, meaning the algorithm can return a false positive.

A Bloom filter consists of an array of size n bits initialized to 0 and k hash functions. Each time an element is added, each hash function maps the element to one of n bits and sets that bit to 1. To check if an element was added the same hash functions are used to find the mapping of the bits. If any of the bits is 0, Bloom filter returns $FALSE$, otherwise $TRUE$.

Assume $k = 5$ and that we insert n elements into a filter of size n. Also assume that we are using perfect independent hash functions.

a) How many bits are set on 1?

b) What is the probability of a false positive?

Sticker album

You own a sticker album that has n stickers. How many stickers do you need on average to fill the album without exchanging stickers with other collectors? Assume that the probability of obtaining each sticker is equal for each sticker. Solve using indicator variables.